

MAU11S02 second Friday quiz ANSWERS

Friday 19/2/21 due 4pm Friday 26/2/21

Rules and procedures.

1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4. *Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Calculate the adjoint, and hence the inverse (no other method), of the following matrix.

$$\begin{bmatrix} 1 & 3 & -3 \\ 1 & 3 & -5 \\ -2 & -8 & 10 \end{bmatrix}$$

Answer.

matrix

$$\begin{bmatrix} 1 & 3 & -3 \\ 1 & 3 & -5 \\ -2 & -8 & 10 \end{bmatrix}$$

determinant -4

adjoint

$$\begin{bmatrix} -10 & -6 & -6 \\ -0 & 4 & 2 \\ -2 & 2 & 0 \end{bmatrix}$$

inverse

$$\begin{bmatrix} 2.5 & 1.5 & 1.5 \\ 0 & -1 & -0.5 \\ 0.5 & -0.5 & -0 \end{bmatrix}$$

Question 2. Let

$$A = \begin{bmatrix} -2 & -4 & 6 \\ 2 & 3 & -3 \\ 1 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & -1 \\ 3 & 5 & 5 \end{bmatrix}$$

Calculate (i) AB (ii) $\det(A)$ (iii) $\det(B)$ (iv) $\det(AB)$.

Answer.

$$(i)AB = \begin{bmatrix} 6 & 18 & 30 \\ -11 & -17 & -8 \\ 8 & 12 & 12 \end{bmatrix}$$

- (ii) Determinant $A = (-2, -4, 6) \cdot (-6, -7, 3) = 58$.
 (iii) Determinant $B = (2, 2, 2) \cdot (15, -13, 4) = 12$.
 (iv) Determinant $AB = (6, 18, 30) \cdot (-108, 68, 4) = 696$.

Question 3. Calculate the (1,1)- and (1,2)- minors of the matrix below.

$$A = \begin{bmatrix} -1 & 3 & -3 & 4 \\ 2 & -2 & 2 & 0 \\ -1 & -1 & 1 & -2 \\ -3 & 4 & -3 & 3 \end{bmatrix}$$

Question 4. Calculate the (1,3)- and (1,4)- minors of the above matrix, and with its four minors calculated, compute its determinant.

Answer. 1 1 minor -4 cofactor -4

1 2 minor 12 cofactor -12

1 3 minor -8 cofactor -8

1 4 minor -4 cofactor 4

determinant 8

Question 5. Here is a system of equations in an unusual form. You can solve it in two ways, either by transforming the system into the usual form and applying Cramer's Rule, or by applying another version of Cramer's Rule. Solve the equations by Cramer's rule in either way.

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ -1 & -1 & 6 \\ 4 & 2 & -17 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

Answer. Take the transpose of everything. We get

$$-1x + -1y + 4z = 0$$

$$0x + -1y + 2z = 1$$

$$3x + 6y + -17z = -2$$

ANSWER

$$\begin{vmatrix} -1 & -1 & 4 \\ 0 & -1 & 2 \\ 3 & 6 & -17 \end{vmatrix} = 1 \quad \begin{vmatrix} 0 & -1 & 4 \\ 1 & -1 & 2 \\ -2 & 6 & -17 \end{vmatrix} = 3 \quad \begin{vmatrix} -1 & 0 & 4 \\ 0 & 1 & 2 \\ 3 & -2 & -17 \end{vmatrix} = 1 \quad \begin{vmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 3 & 6 & -2 \end{vmatrix} = 1$$

$$x = \frac{3}{1} = 3 \quad y = \frac{1}{1} = 1 \quad z = \frac{1}{1} = 1$$