MAU11S02 first Monday quiz, week 2, Answers
Monday 8/2/21 due 4pm Monday 15/2/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Solve by Cramer's Rule (no other method)

$$-1x + 1y = -1; \quad -3x + 5y = -1$$

$$\begin{vmatrix} -1 & 1 \\ -3 & 5 \end{vmatrix} = -2, \quad \begin{vmatrix} -1 & 1 \\ -1 & 5 \end{vmatrix} = -4, \quad \begin{vmatrix} -1 & -1 \\ -3 & -1 \end{vmatrix} = -2,$$

$$x = 2, \quad y = 1$$

Question 2. Calculate the adjoint matrix, and hence invert

$$\begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$\left(\frac{-1}{2}\right) \begin{bmatrix} 5 & -1 \\ 3 & -1 \end{bmatrix}$$

Question 3. This and the next question will be to solve the linear system

$$\begin{aligned} 1x + -2y + 6z &= 18 \\ 3x + -6y + 17z &= 51 \\ -2x + 6y + -18z &= -50 \end{aligned}$$

using Cramer's Rule (no other method). Writing this in the form $Px + Qy + Rz = S$, calculate the determinant of the matrix with columns P, Q, R, and then calculate the determinant of the matrix with columns S, Q, R. Hence compute x.

Question 4. Calculate the other two determinants arising in Cramer's Rule (3×3 case) and hence calculate y and z.

$$\begin{vmatrix} 1 & -2 & 6 \\ 3 & -6 & 17 \\ -2 & 6 & -18 \end{vmatrix} = 2 \quad \begin{vmatrix} 18 & -2 & 6 \\ 51 & -6 & 17 \\ -50 & 6 & -18 \end{vmatrix} = 8 \quad \begin{vmatrix} 1 & 18 & 6 \\ 3 & 51 & 17 \\ -2 & -50 & -18 \end{vmatrix} = 4 \quad \begin{vmatrix} 1 & -2 & 18 \\ 3 & -6 & 51 \\ -2 & 6 & -50 \end{vmatrix} = 6$$

$$x = \frac{8}{2} = 4 \quad y = \frac{4}{2} = 2 \quad z = \frac{6}{2} = 3$$

Question 5. A parallelopiped is a solid figure analogous to a parallelogram. It has six parallel faces (for example, a cube). If it has one corner at the origin adjacent to three corners P, Q, R, then the other 4 corners are various sums of these points. The volume of the parallelpiped is $\pm\vec{OP} \cdot (\vec{OQ} \times \vec{OR})$, and the volume of the tetrahedron $OPQR$ is one-sixth of this. Calculate the volume of $OPQR$ given $P = (2, 1, 2), Q = (8, 5, 10), R = (3, 1, -1)$.

$$\begin{vmatrix} 2 & 1 & 2 \\ 8 & 5 & 10 \\ 3 & 1 & -1 \end{vmatrix} = -6$$

so the required volume is $6/6 = 1$.