MA1S12 Group A2 Quiz 09 11am 29/3/18 ANSWERS

Instructions. Attempt 3 questions as usual. This is the last written quiz.

(1) (i) Tabulate $B(10, .6)$ $p_0, \ldots, p_{10}$, each to 4 decimal places. (ii) Calculate Prob$(X \le 3)$, where $X \sim B(10, 0.6)$.

Answer.

0	1	2	3	4	5	6	7	8	9	10
0.0002	0.0024	0.0159	0.0637	0.1672	0.3010	0.3762	0.3225	0.1814	0.0605	0.0060

(ii) .0821.

(2) From the Central Limit Theorem, if $X \sim B(n,p)$ (binomial), then $(X-np)/(\sqrt{np(1-p)}) \sim N(0,1)$, approximately. Use this to give an approximation to Prob$(X \le 3)$, where $X \sim B(10, 0.6)$.

Answer.

$$\text{Prob}\left(\frac{X-6}{\sqrt{2.4}} \le \frac{3-6}{\sqrt{2.4}} = -\frac{3}{1.5491} = -1.9366\right) \approx 0.362$$

(3) Calculate (to 4 decimal places) the sample mean (average) $\overline{X}$, the sample variance S^2, and the sample standard deviation S, of the following numbers

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3.00 3.11 1.54 3.46 2.39 3.15 3.27 3.02 0.56
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Answer.

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Sample mean 2.6111 sample variance 0.9270 sdev 0.9628
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(4) Assuming that these numbers are iid $N(\mu, 1)$, so σ is given but not μ, give a 95% symmetric confidence interval for μ.

Answer.

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Average of 9: 2.6111; assumed s dev 1.0000
percentage point 1.9600
[1.9578, 3.2644]
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(5) Assuming that these numbers are iid $N(\mu, \sigma^2)$ where neither μ nor σ is known, give a 95% symmetric confidence interval for μ.

Answer.

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Average of 9: 2.6111; sample s dev 0.9628
8 degrees of freedom, percentage point 2.306
percentage point 2.3060
[1.8710, 3.3512]
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(6) (Later.) Assuming that these numbers are iid $N(\mu, \sigma^2)$ where neither μ nor σ is known, give a 95% 2-sided confidence interval for σ^2 and for σ.

Answer. Sample size 9, sample variance 0.9270, chi-squared 8 degrees of freedom, 2.5 percentage points 2.1797, 17.535, Confidence interval $[8 \times .9270/17.535, 8 \times .9270/2.1797]$ $[.4229, 3.4023]$

(7) (Later.) A bag contains red and white snooker balls. Originally there were exactly twice as many red as white balls but it is believed that the ratio is lower than that.

In order to test this hypothesis, rather than counting the balls, it is decided to take out, randomly, and return, a ball from the bag 15 times, and count how many were red.

(i) By direct calculation with $B(15, 2/3)$, find the largest a in $\{0, \ldots, 15\}$ such that $\{0, \ldots, a-1\}$ has probability $\leq 10\%$. The Normal approximation is too inaccurate here. Calculate the probabilities directly. ($p_0 = p_1 = 0$ to 4 decimal places).

(ii) The hypothesis being tested is that the ratio of red to white balls in the bag is less than $2:1$. State the null hypothesis and give the rejection region at 10% significance. Suppose the outcome was 6 red balls out of 15. Do you reject or accept the null hypothesis at 10% significance?

Answer. (i) $p_0 + \ldots + p_6 = .0616$, $+p_7 = .1764$. So $a = 7$.

(ii) Null hypothesis is that the ratio is exactly $2:1$. The 10% rejection region is $\{0, \ldots, 6\}$, so at that significance level the null hypothesis is rejected.