**Rules and procedures: this week is different.**

**(1)** A factory has three production lines, $A, B, C$ with relative rates $1 : 2 : 2$ of production. $D$ is the event that an item is defective. The conditional probabilities of these events are: $P(D|A) = 0.5$, $P(D|B) = 0.2$, and $P(D|C) = 0.6$. Calculate, to 4 decimal places, $P(D)$ and $P(X|D)$ for $X = A, B, C$.

**Answer.**

|            | $A$    | $B$    | $C$    | $D$  |
|------------|--------|--------|--------|------|
| $P(X)$     | .2     | .4     | .4     |      |
| $P(D|X)$   | .5     | .2     | .6     |      |
| $P(D \cap X)$ | .1  | .08    | .24    |      |
| $P(D)$     |        |        |        | .42  |
| $P(X|D)$   | .2380  | .1904  | .5714  |      |

**(2)** Give the probability distribution for the outcomes, 2 to 12, on throwing two fair dice. Give the mean and variance.

**Answer.** Probabilities, $2 \ldots 12$:

1/36 2/36 3/36 4/36 5/36 6/36 5/36 4/36 3/36 2/36 1/36

mean $(2*1 + 3*2 + 4*3 + 5*4 + 6*5 + 7*6 + 8*5 + 9*4 + 10*3 + 11*2 + 12*1)/36 = 7$

Variance $(5^2*1 + 4^2*2 + 3^2*3 + 2^2*4 + 1^2*5)*2/36 = 5.8333$

**(3)** Cars arrive at a traffic lights at random under a Poisson distribution. They arrive at the rate 5/minute. The green period is 20 seconds long and the red is 40. In a green period, up to 6 cars can get through. Suppose that the lights turn red at time $T$, and 2 cars are left waiting. What is the probability that there will be 2 cars waiting (i) when the lights next turn green, 20 seconds after $T$, and (ii) when they next turn red, 60 seconds after $T$?

**Answer.** (i) $\mu t = 5/3$. No cars arrive, $p_0 = .1888$
(ii) $\mu t = 5$, 6 arrive, 6 get through: $p_6 = e^{-5} 5^6 / 6! = .1453$.

**(4)** Under the same conditions as (ii) above, what is the probability that there will be no cars waiting when they next turn red, 60 seconds after $T$?

**Answer.** $\mu t = 5$, 2 at time $T$, up to 6 got through, up to 4 arrived: $e^{-5}(1 + 5 + 25/2 + 125/6 + 625/24) = .4380$

**(5)** Under the same conditions as above, *except* that the red and green periods are both 30 seconds long, and the lights turn green at time $T$, suppose there are two cars left waiting when the lights turn green. What is the probability that there will be three cars left waiting when the lights turn green again, one minute after $T$?

**Answer.** Both period have arrivals under Poisson (2.5) and again up to 6 cars can get through. The cases are:

2 at $T$, up to 4 arrive, 0 at $T + 30$, 3 arrive:
$(p_0 + p_1 + p_2 + p_3 + p_4) * p3 = .1898$
5 arrive, 1 at $T + 30$, 2 arrive: $p_5 p_2 = .0168$
6 arrive, 2 at $T + 30$, 1 arrives: $p_6 p_1 = .0055$
7 arrive, 3 at $T + 30$, none arrive: $p_7 p_0 = .0008$
$.1898 + .0168 + .0055 + .0008 = .2129$.