MA1S12 Group A2 Quiz 06 11am 8/3/18 ANSWERS

Rules and procedures: UNCHANGED

(1) Calculate a right-handed orthonormal basis X_1, X_2, X_3 where X_3 is in the direction $\vec{ON}$, where N is $(12, 5, 0)$.

Answer.

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 12   5  0     12   5   0
x 0   0  1   x  5 -12   0
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  5 -12  0      0   0  -169
```

Answer.

$$\begin{bmatrix} 5/13 & 0 & 12/13 \\ -12/13 & 0 & 5/13 \\ 0 & -1 & 0 \end{bmatrix}$$

(2) Hence or otherwise compute the matrix (in standard coordinates) for perpendicular projection onto the plane $\{X : \vec{ON} \cdot \vec{OX} = 0\}$.

Answer.

$$\begin{bmatrix} 5/13 & 0 & 12/13 \\ -12/13 & 0 & 5/13 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5/13 & -12/13 & 0 \\ 0 & 0 & -1 \\ 12/13 & 5/13 & 0 \end{bmatrix} = \begin{bmatrix} 25/169 & -60/169 & 0 \\ -60/169 & 144/169 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) Using (1) or otherwise, compute the matrix for rotation through 45° around the axis ON (same N as above).

Answer.

$$\begin{bmatrix} 5/13 & 0 & 12/13 \\ -12/13 & 0 & 5/13 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5/13 & -12/13 & 0 \\ 0 & 0 & -1 \\ 12/13 & 5/13 & 0 \end{bmatrix} =$$
$$\begin{bmatrix} \frac{25}{169\sqrt{2}} + \frac{144}{169} & -\frac{60}{169\sqrt{2}} + \frac{60}{169} & \frac{5}{13\sqrt{2}} \\ -\frac{60}{169\sqrt{2}} + \frac{60}{169} & \frac{144}{169\sqrt{2}} + \frac{25}{169} & -\frac{12}{13\sqrt{2}} \\ -\frac{5}{13\sqrt{2}} & \frac{12}{13\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(4) Let A be the following matrix:

$$\begin{bmatrix} 17 & -12 \\ 24 & -17 \end{bmatrix}$$

Calculate eigenvalues and corresponding eigenvectors for A.

Answer.

$$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 17 & -12 \\ 24 & -17 \end{bmatrix}$$

(5) Hence calculate e^{At}.
Answer.

corrected

$$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 9e^t - 8e^{-t} & -6e^t + 6e^{-t} \\ 12e^t - 12e^{-t} & -8e^t + 9e^{-t} \end{bmatrix}$$