MA1S12 Group A2 Quiz 04 11am 15/2/18 ANSWERS

Rules and procedures: UNCHANGED

(1) Let A be the matrix

$$\begin{bmatrix} 0 & -2 & -6 & 0 & -2 \\ 0 & 0 & 0 & -2 & -6 \\ -1 & 2 & 2 & -1 & -3 \\ -1 & 2 & 2 & -3 & -9 \\ -2 & 2 & -2 & -5 & -17 \end{bmatrix}$$

Bring to RREF, and hence give bases for its row and column spaces.

Answer.

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 2 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row space basis: rows 1,2,3 of RREF Column space: Columns 1,2,4 of original

(2) Give a basis for the kernel (nullspace) of the above matrix A.

Answer. Columns of matrix below

$$\begin{bmatrix} -4 & -2 \\ -3 & -1 \\ 1 & 0 \\ 0 & -3 \\ 0 & 1 \end{bmatrix}$$

(3) Calculate the perpendicular projection of the point $(4, 1, 3)$ onto the line through O and $(3, 4, 12)$

Answer. $(4, 1, 3) \cdot (3, 4, 12)/(9 + 16 + 144)(3, 4, 12) = (12/13, 16/13, 48/13)$.

(4) Hence or otherwise calculate the perpendicular projection of the same point onto the plane through O, $(4, -3, 0)$ and $(40, 45, -25)$.

Answer. $(4, 1, 3) - (12/13, 16/13, 48/13) = (40/13, -3/13, -9/13)$.

(5) Calculating the effect of rotating $(4, 1, 3)$ through $45°$ around the axis OV, where $V = (3, 4, 12)$.

$Z = (12/13, 16/13, 48/13), Y = (40/13, -3/13, -9/13), W = V \times Y = (0, 3, -1)$. Answer: $(12 + 20\sqrt{2}, 16 + 18\sqrt{2}, 48 - 11\sqrt{2})$.