MA1S12 Group A2 Quiz 03 11am 8/2/18 ANSWERS

Rules and procedures: UNCHANGED

(1) Calculate the following determinant

$$\begin{vmatrix} 2 & 0 & -4 & 2 \\ 3 & 1 & -7 & -1 \\ 3 & -3 & -2 & 16 \\ -3 & 1 & 6 & -8 \end{vmatrix}$$

by cofactor expansion along the third column. (see overpage).

Answer.

$$\begin{vmatrix} 3 & 1 & -1 \\ 3 & -3 & 16 \\ -3 & 1 & -8 \end{vmatrix} = 6 \quad \begin{vmatrix} 2 & 0 & 2 \\ 3 & -3 & 16 \\ -3 & 1 & -8 \end{vmatrix} = 4 \quad \begin{vmatrix} 2 & 0 & 2 \\ 3 & 1 & -1 \\ -3 & 1 & -8 \end{vmatrix} = -2 \quad \begin{vmatrix} 2 & 0 & 2 \\ 3 & 1 & -1 \\ 3 & -3 & 16 \end{vmatrix} = 2$$

Determinant $(1)(-4)(6) + (-1)(-7)(4) + (1)(-2)(-2) + (-1)(6)(2) = -4$

(2) Calculate the same determinant by reducing to upper triangular form.

Answer.

$$\begin{bmatrix} 2 & 0 & -4 & 2 \\ 0 & 1 & -1 & -4 \\ 0 & -3 & 4 & 13 \\ 0 & 1 & 0 & -5 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & -4 & 2 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & -4 & 2 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & -4 & 2 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

0 swaps, Determinant is $-4/1$

(3) (i) Let A be the matrix

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & -4 & 6 \\ 2 & -6 & 10 \end{bmatrix}$$

Calculate $\det A$ and $\det \mathrm{Adj}(A)$. (There are short-cuts.)

Answer.

```
matrix
        1        -2         4
        2        -4         6
        2        -6        10
determinant -4
adjoint
       -4        -4         4
       -8         2         2
       -4         2         0
Determinant of adjoint: 16
```

(ii) What is $\det \operatorname{Adj}(A)$ as a function of $\det(A)$ for general $A_{n\times n}$? Note: $\det cA = c^n \det A$.
Answer. $\det(A)^{n-1}$.
(4) Determine if the columns of the following matrix are linearly independent.

$$\begin{bmatrix} 0 & 2 & 4 \\ 1 & -3 & -4 \\ -2 & 6 & 7 \end{bmatrix}$$

Answer. RREF is the identity matrix. Linearly independent.
(5) Determine if the columns of the following matrix are linearly independent.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 3 & 0 & 6 \end{bmatrix}$$

Answer.

```
1  0  2 rref.  Linearly dependent.
0  1  1
0  0  0
```