MA1S12 Group A3 Quiz 02 4pm 1/2/18 ANSWERS

Rules and procedures.
1. You are welcome to consult notes and collaborate during tutorials. **2.** You should **attempt 3 questions**. If more are attempted, then the first three attempted will be marked. All questions will be marked out of 20, so all quizzes will contribute 60 marks. **3.** Answer sheets will be collected at the end of each tutorial. **4.** Answer sheets will not be accepted at any other time or in any other way. **5.** Answer sheets will be marked and returned the following week. **6.** Answers to each quiz will be distributed the following week. **7.** Show all your work. **8.** If a particular method is stipulated, such as Cramer's Rule, you must use it. No credit will be given for the wrong method. **9.** Course work will contribute 20% to your overall mark.

(1) Calculate the $(1,1)$ and the $(1,2)$ minors of the following determinant.

$$\begin{vmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & 2 \\ 3 & -4 & -5 & -1 \\ 0 & 2 & 1 & -3 \end{vmatrix}$$

(2) Calculate the $(1,3)$ and $(1,4)$ minors of the above determinant. Hence calculate the determinant by cofactor expansion.

Answer.

$$\begin{vmatrix} 0 & 1 & 2 \\ -4 & -5 & -1 \\ 2 & 1 & -3 \end{vmatrix} = -2 \quad \begin{vmatrix} 0 & 1 & 2 \\ 3 & -5 & -1 \\ 0 & 1 & -3 \end{vmatrix} = 15 \quad \begin{vmatrix} 0 & 0 & 2 \\ 3 & -4 & -1 \\ 0 & 2 & -3 \end{vmatrix} = 12 \quad \begin{vmatrix} 0 & 0 & 1 \\ 3 & -4 & -5 \\ 0 & 2 & 1 \end{vmatrix} = 6$$

Determinant $(1)(1)(-2) + (-1)(-2)(15) + (1)(-1)(12) + (-1)(3)(6) = -2$

(3) Solve the following system using Cramer's Rule (no other method).

$$\begin{bmatrix} 2 & -2 & -6 \\ -1 & 3 & 5 \\ -3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -14 \\ 15 \\ 9 \end{bmatrix}$$

Answer.

$$a = \begin{vmatrix} 2 & -2 & -6 \\ -1 & 3 & 5 \\ -3 & 1 & 5 \end{vmatrix} = -8 \quad b = \begin{vmatrix} -14 & -2 & -6 \\ 15 & 3 & 5 \\ 9 & 1 & 5 \end{vmatrix} = -8 \quad c = \begin{vmatrix} 2 & -14 & -6 \\ -1 & 15 & 5 \\ -3 & 9 & 5 \end{vmatrix} = -16$$

$$d = \begin{vmatrix} 2 & -2 & -14 \\ -1 & 3 & 15 \\ -3 & 1 & 9 \end{vmatrix} = -16 \quad x = \frac{-8}{-8} \quad y = \frac{-16}{-8} \quad z = \frac{-16}{-8}$$

(4) Calculate the adjoint of the matrix below, and *hence* compute its inverse.

$$\begin{bmatrix} 2 & -2 & -6 \\ -1 & 3 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

Answer.

$$\left(\frac{1}{-8}\right)\begin{bmatrix} 10 & 4 & 8 \\ -10 & -8 & -4 \\ 8 & 4 & 4 \end{bmatrix}$$

(5) Calculate the following determinant by cofactor expansion *along the second row*:

$$\sum_j a_{2j}(-1)^{2j}\text{minor}_{2j}(A)$$

$$\begin{vmatrix} -2 & -4 & 12 \\ 2 & 5 & -14 \\ -1 & -2 & 7 \end{vmatrix}$$

Answer.

$$\begin{vmatrix} -4 & 12 \\ -2 & 7 \end{vmatrix} = -4 \quad \begin{vmatrix} -2 & 12 \\ -1 & 7 \end{vmatrix} = -2 \quad \begin{vmatrix} -2 & -4 \\ -1 & -2 \end{vmatrix} = 0$$

$$\text{Determinant } (-1)(2)(-4) + (1)(5)(-2) + (-1)(-14)(0) = -2$$