

Faculty of Engineering, Mathematics and Science School of Mathematics

JF Science			Trinity Term 2022
Mathematics U11S02 —	Mathematics for s	cientists	Questions 1-4
Tuesday, 3 May 2022	RDS main	9.30–12	2.30
Profs. Miria	m Logan and Colr	n Ó Dúnlai	ng

Instructions to Candidates:

Attempt 6 questions, 3 from each section. Show all work. Remember to fold down and glue the flap on every answer booklet.

You may not start this examination until you are instructed to do so by the Invigilator.

1. (a) (6 points). Calculate the adjoint, and hence the inverse (no other method), of

$$\left[\begin{array}{rrrrr} 1 & 2 & -1 \\ 1 & 3 & -2 \\ -3 & -4 & 5 \end{array}\right]$$

(b) (6 points). Calculate the following determinant, by cofactor expansion along the second column.

- (c) (4 points). Compute the above determinant, again, by reducing to upper triangular form.
- (d) (4 points). Compute bases for the row space, column space, and kernel (nullspace) of the following matrix

$$\begin{bmatrix} -1 & 3 & 6 & 3 \\ -3 & 8 & 15 & 7 \\ -2 & 7 & 15 & 8 \end{bmatrix}$$

- (a) (6 points) Construct a right-handed orthonormal basis for ℝ³ whose third vector X₃ is in the direction (4, 3, 0).
 - (b) (2 points) Give the matrix A' for perpendicular projection on the xy-plane.
 - (c) (6 points) Hence or otherwise calculate the matrix A of the perpendicular projection of points onto the plane 4x + 3y = 0.
 - (d) (6 points) Calculate the point obtained by rotating the point (1, 2, 3) through 45° around the axis through (4, 3, 0).
- 3. (a) (6 points) Given the following list of data points in \mathbb{R}^2

$$(-1,3)$$
 $(0,4)$ $(1,1)$ $(2,1)$

calculate the line y = mx + c best fitting this data (least squared error). Apply the usual formula and show all work.

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- (b) (8 points) Given the same four data points, the (quadratic) curve $y = ax^2+bx+c$ best fitting the data, using the same formula and showing all work.
- (c) (6 points) Given the same four data points, calculate the curve $y = ax^2 + b$ best fitting the data.
- (a) (4 points) Calculate the sample mean (average) and sample standard deviation of the following numbers

3.54 -1.88 -0.21 -4.78 4.01 6.25 0.32 7.35 2.14 2.87

- (b) (6 points) The above numbers are distributed $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Using Student's *t*-distribution, give a 95% symmetric 2-sided confidence interval for μ .
- (c) (4 points) Suppose that $X \sim B(160, 0.1)$. Using the normal approximation, with the continuity correction, calculate the probability that $X \ge 22$.
- (d) (6 points) It is reckoned that the worldwide proportion of left-handed people is between 10% and 12%. In a certain module there are 160 students enrolled. Suppose that they were polled and that 22 of them reported they were left-handed. Applying a significance test, take the null hypothesis to say that the probability of a student being left-handed is 10% and the alternative hypothesis to say that the probability is higher than that. Decide whether the figure of 22 is significant (supports the alternative hypothesis) at the 5% level.