MAU11602 fifth quiz, week 11
Thu 15/04/21 due 11am Friday 23/4/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. A property P of partial recursive functions $\phi_m()$ is any subset of the set of all partial recursive functions ϕ_m. P is *nontrivial* if P and its complement are nonempty.

Implicitly, if $\phi_m \in P$ and $\phi_{m'} = \phi_m$, then $\phi_{m'} \in P$.

Prove (Rice's Theorem) that if P is nontrivial then P is not recursive, meaning

$$\{m : \phi_m \in P\}$$

is a nonrecursive subset of $\mathbb{N}$.

Question 2. Is the set $\{m : \ \phi_m \quad \text{is recursive}\}$ recursive? Give reasons.

Question 3. Show that the sets $A = \{m : \ \phi_m(n) \downarrow 0 \text{ for all } n\}$, and $B = \{m : \ \phi_m(n) \downarrow 1 \text{ for all } n\}$ are recursively inseparable.

Question 4. Let X and Y be recursively inseparable sets, where one of them is recursive. Prove that $X \cap Y \neq \emptyset$.

Question 5. The set of theorems of PA is recursively enumerable (given a suitable encoding of the formulae of PA as bitstrings or numbers). Prove that the set of formulae of PA which are *not* theorems of PA is not even recursively enumerable.