# MAU11602 fourth quiz ANSWERS
# Thu 1/04/21 due 11am Wednesday 14/4/21

**Rules and procedures.**
**1.** Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. Suppose that $K$ is a first-order theory with at least one function letter, and that $K$ has a model. Prove that $K$ has a countable model.
Answer. Since $K$ has at least one function letter, the set of all closed terms is countable. Since $K$ has a model, it is consistent, and has a model whose domain is the set of all closed terms of an extension of $K$.

Question 2. Define 2 as $s(s(0))$ and $4 = s(s(s(s(0))))$. Give a formal annotated proof that $2 + 2 = 4$.
Answer.

```
1. 2 + 2 = s(s(0)) + s(s(0)) (definition)
2. s(s(0)) + s(s(0)) = s(s(s(0)) + s(0)) (Ax 6)
  = s(s(s(s(0+0))))  (Ax 6)
  = s(s(s(s(0)))) (Ax 5)
  = 4.
```

Question 3. Define 1 as $s(0)$. Give a formal annotated proof that $x + 1 = s(x)$.
Answer.

```
1. x+1 = x+s(0) (Definition)
  = s(x+0) (Axiom 6)
  = s(x) (Axiom 5).
```

Question 4. Prove by induction on $x$ that $0x = 0$. This is 19.11 in the web notes, and you may use anything result up to 19.10 in your proof. Your proof should be well-annotated but need only prove the base $P(0)$ and $P(s(x))$ assuming $P(x)$.
Answer.

```
1. 00 = 0 (Axiom 7): P(0)
2. 0x=0  (inductive hypothesis)
3. 0s(x) = 0x + 0 (Axiom 8)
    = 0 + 0 ( ind hyp )
    = 0 (Ax 5).  P(s(x)).
```

Question 5. Prove that $x(y+z) = (xy) + (xz)$. (See 19.16, which is very similar). You may use results up to 19.14. Use induction on $x$: $P(0)$, $P(s(x))$. Axiom 4 need not be invoked explicitly.

In your answer there will be terms rearranged, invoking commutativity and associativity of addition. In the notes, 19.16, there was cutting of corners. But your answer should explicitly justify rearrangement of terms.

Answer.

```
1. 0(y+z) = 0 (19.11)
2. 0y + 0z = 0 + 0 = 0 (19.11, Ax 5)
3. 0(y+z) = 0y + 0z  (1,2,equality reasoning)
    P(0)
4. x(y+z) = xy + xz (inductive hypothesis)
5. s(x)(y+z) = x(y+z) + (y+z) (19.12)
   = (xy+xz) + (y+z) (ind. hypoth)
   = (xz+xy) + (y+z) (commut)
   = ((xz + xy) +y ) + z (assoc)
   = (xz + (xy+y) ) + z (assoc)
   = (xz + s(x)y) + z (19.12)
   = (s(x)y + xz) + z (commut)
   = s(x)y + (xz+z) (assoc)
   = s(x)y + s(x) z
     P(s(x))
```