MAU11602 third quiz, week 6
Thu 11/03/21 due 11am Friday 26/03/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. Granted $\exists x_i A(x_i)$ means $\neg(\forall x_i(\neg A(x_i)))$, give a simple definition of $I, \sigma \models \exists x_i A(x_i)$.

Question 2. In a theory of groups, take f_1 to mean multiplication, f_2 means inverse, a_1 identity, and P_1 equality.

Give closed formulae using the strict language of G which express: (i) multiplication is associative, (ii) a_1 is left identity, (iii) a_1 is a right identity, (iv) f_2 is a left inverse, and (v) f_2 is a right inverse.

Question 3. Give an example where $A(t) \implies \exists x_i A(x_i)$ is false. Hint: $(\forall x_1 \exists x_2(x_1 \neq x_2) \implies \exists x_2(x_2 \neq x_2))$.

Question 4. With the above formal Group Theory, we construct an interpretation M whose domain is the set of 2×2 matrices with integer entries and determinant ± 1; f_1^M is matrix multiplication, f_2^M is matrix inverse, and a_1^M is the 2×2 identity matrix I.

Let

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 11 & -20 \\ 6 & -11 \end{bmatrix}.$$

Note: there were two mistakes in this question. The domain was enlarged to make C, whose determinant is -1, correct. The matrix B was corrected by changing a 5 to 2, making its determinant 1.

Evaluate the truth

(i) Of the formula $P_1(f_1(x_1, x_2), x_3)$ under the snapshot $A, A, A, \ldots$
(ii) Of the same formula under snapshot $A, B, I \ldots$
(iii) Of the same formula under snapshot $A, A, I \ldots$
(iv) Of the same formula under snapshot $(C, C, I, \ldots$

Question 5. (i) Correct or incorrect? for any interpretation I of a theory K, and any formula A of K, either $I \models A$ or $I \models \neg A$. (ii) Correct or incorrect? for any interpretation I of a theory K, and any *closed* formula A of K, either $I \models A$ or $I \models \neg A$. Give reasons for your answers.