MAU11602 second quiz ANSWERS
Thu 25/02/21 due 11am Friday 05/03/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. Construct a CNF for the following truth-table.

W	X	Y	Z	
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1

W	X	Y	Z	
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Answer.

$$W X \overline{Y}\, Z,\ W \overline{X}\, Y \overline{Z},\ \overline{W}\, X Y \overline{Z},\ \overline{W}\, \overline{X}\, \overline{Y}\, Z,\ \overline{W}\, \overline{X}\, \overline{Y}\, \overline{Z}$$

Question 2. A kind of 'accelerated resolution' might be to infer $C \vee D$ from $C \vee X \vee Y, D \vee \overline{X} \vee \overline{Y}$.

Why is this *completely wrong*?

Answer. It is wrong to suggest that if

$$f \models C \vee X \vee Y \quad \text{and} f \models C \vee \overline{X} \vee \overline{Y}$$

Then

$$f \models C \vee D$$

For example, if C and D are empty and f makes X true and Y false, then it satisfies the first two clauses but $C \vee D$ is the empty clause. So one cannot infer the latter from the former.

Question 3. Use resolution to refute

$$U\overline{W},\ \overline{U}\, W,\ U\overline{Y},\ \overline{U}\, Y,\ V,\ \overline{V}\, XY,\ V\overline{X}\, Y,\ V X \overline{Y},$$
$$\overline{V}\, \overline{X}\, \overline{Y},\ W\overline{X},\ \overline{W}\, X$$

Answer.

$$U\overline{W},\ \overline{U}\,Y \to \overline{W}\,Y \quad \overline{U}\,W,\ U\overline{Y} \to W\overline{Y} \quad W\overline{X}, \overline{W}\,Y \to \overline{X}\,Y$$
$$\overline{W}\,X, W\overline{Y} \to X\overline{Y} \quad V,\ \overline{V}XY \to XY, \quad V,\ \overline{V}\,\overline{X}\,\overline{Y} \to \overline{X}\,\overline{Y}$$
$$XY,\ \overline{X}\,Y \to Y \quad X\overline{Y}, \overline{X}\,\overline{Y} \to \overline{Y}$$
$$Y, \overline{Y} \to \square$$

Question 4. Use resolution to refute

$$UVX,\ \overline{U}\,V\overline{X},\ \overline{U}\,\overline{V}\,X,\ U\overline{V}\,\overline{X},\ \overline{U}\,W,\ U\overline{W},\ VWY,\ \overline{V}\,\overline{W}\,Y,$$
$$V\overline{W}\,\overline{Y},\ \overline{V}\,W\overline{Y},\ XY,\ \overline{X}\,\overline{Y}$$

Answer.

$$UVX,\ \overline{U}\,V\overline{X},\ \overline{U}\,\overline{V}\,X,\ U\overline{V}\,\overline{X},\ \overline{U}\,W,\ U\overline{W}$$
$$VWY,\ \overline{V}\,\overline{W}\,Y,\ V\overline{W}\,\overline{Y},\ \overline{V}\,W\overline{Y},\ XY,\ \overline{X}\,\overline{Y}$$

$$UVX,\ \overline{U}\,W \mapsto VWX$$
$$\overline{U}\,V\overline{X},\ U\overline{W} \mapsto V\overline{W}\,\overline{X}$$
$$\overline{U}\,\overline{V}\,X,\ U\overline{W} \mapsto \overline{V}\,\overline{W}\,X$$
$$U\overline{V}\,\overline{X},\ \overline{U}\,W \mapsto \overline{V}\,W\overline{X}$$
$$VWX,\ \overline{V}\,W\overline{Y} \mapsto WX\overline{Y}$$
$$V\overline{W}\,\overline{X},\ \overline{V}\,\overline{W}\,Y \mapsto \overline{W}\,\overline{X}\,Y$$
$$\overline{V}\,\overline{W}\,X,\ V\overline{W}\,\overline{Y} \mapsto \overline{W}\,X\overline{Y}$$
$$\overline{V}\,W\overline{X},\ VWY \mapsto W\overline{X}\,Y$$
$$WX\overline{Y},\ XY \mapsto WX$$
$$\overline{W}\,\overline{X}\,Y,\ \overline{X}\,\overline{Y} \mapsto \overline{W}\,\overline{X}$$
$$\overline{W}\,X\overline{Y},\ XY \mapsto \overline{W}\,X$$
$$W\overline{X}\,Y,\ \overline{X}\,\overline{Y} \mapsto W\overline{X}$$
$$WX,\ \overline{W}\,X \mapsto X$$
$$W\overline{X},\ \overline{W}\,\overline{X} \mapsto \overline{X}$$
$$X,\ \overline{X} \mapsto \square$$

Question 5. Using any result up to 10.12, but nothing later than that, prove

$$A \vdash_{\mathrm{SC}} \neg\neg A$$

Answer.

1. $\neg\neg\neg A \vdash \neg A$ (10.11)
2. $(\neg\neg\neg A) \Rightarrow \neg A$ (1, D. thm)
3. A given
4. $A \Rightarrow ((\neg\neg\neg A) \Rightarrow A)$ Axiom I.
5. $(\neg\neg\neg A) \Rightarrow A$ (3,4,MP).
6. $((\neg\neg\neg A) \Rightarrow \neg A) \implies ((\neg\neg\neg A) \Rightarrow A) \Rightarrow \neg\neg A)$ (Axiom III).
7. $((\neg\neg\neg A) \Rightarrow A) \Rightarrow \neg\neg A)$ (2,6,MP)
8. $\neg\neg A$ (5,7,MP).