MAU11602 first quiz, week 2 ANSWERS
Thu 11/02/21 due 11am Friday 19/02/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

All turing machines must conform to the pattern required for the turinginC program. Also, if and when a machine terminates, the square being scanned should contain the leftmost nonblank tape symbol (blank if the tape is blank).

Question 1. There is a machine for incrementing binary strings on the web page. Modify it so that it loops on empty input.

Answer

```
q0 B B R q0
q0 0 0 R q1
q0 1 1 R q1
q1 0 0 R q1
q1 1 1 R q1
q1 B B L q2
q2 1 0 L q2
q2 0 1 L q3
q2 B 1 L q3
q3 0 0 L q3
q3 1 1 L q3
q3 B B R q4
```

```
input: 1101
output: 1110

input: 0
output: 1

input: 00
output: 01
```

```
input: 01
output: 10

input:
output: loops

input: 1111
output: 10000

input: 1100
output: 1101

input: 0100
output: 0101

input: 1110000110111
output: 1110000111000
```

Question 2. Present a Turing machine which, on input x (a bitstring), halts with $|x| \mod 2$ (0/1) on the tape.

Answer

```
q0 B 0 L q3
q0 0 B R q2 q1 even length so far
q0 1 B R q2 q2 odd
q1 0 B R q2 q1 even
q1 1 B R q2
q2 0 B R q1 odd
q2 1 B R q1
q1 B 0 L q3 even length
q2 B 1 L q3 odd
q3 B B R q4 scanning 0 or 1

% tm lenmod2.tu sample-bitstrings

input: 1101
output:    0

input: 0
output: 1
```

```
input: 00
output:  0

input: 01
output:  0

input:
output: 0

input: 1111
output:    0

input: 1100
output:    0

input: 0100
output:    0

input: 1110000110111
output:             1
```

Question 3. Present a Turing machine which halts with empty tape if the input x has the same number of occurrences of 0 as of 1, and loops otherwise.

Answer

```
    Halt if 0/1 balance, loop otherwise
    Tape contents on halting are irrelevant.
q0 0 x R q1    look for a 1
q0 1 y R q3    look for a 0

q1 0 0 R q1    looking for 1
q1 x x R q1
q1 y y R q1
q1 1 y L q2    found a 1.
q1 B B R q5    burst -- excess of 0

q2 0 0 L q2    move to left end
q2 1 1 L q2    move to left end
q2 x x L q2    move to left end
q2 y y L q2    move to left end
q2 B B R q4    to left end, move right
```

```
q3 1 1 R q3    looking for 0
q3 x x R q3
q3 y y R q3
q3 0 x L q2    found a 0. Move left
q3 B B R q5    burst -- excess of 1

q4 x x R q4
q4 y y R q4
q4 0 x R q1
q4 1 y R q3   q4 B: halt

q5 0 0 R q5
q5 1 1 R q5
q5 x x R q5
q5 y y R q5
q5 B B R q5

tm balance.tu sample-bitstrings

input: 1101
output: loops

input: 0
output: loops

input: 00
output: loops

input: 01
output: xy

input:
output:

input: 1111
output: loops

input: 1100
output: yyxx

input: 0100
output: loops

input: 1110000110111
output: loops
```

Question 4 Present a Turing machine which on input x halts with x^R on the tape, where x^R is the reversal of x.

Answer

```
   Form the reversal of the
   input string and clean up, preferably
   with the rw head at the leftmost
   square of the reversed string.


q0 0 0 R q0
q0 1 1 R q0
q0 B B L q1  found right end
q1 x x L q1  seek left
q1 y y L q1
q1 X X L q1
q1 Y Y L q1
q1 0 x R q2  until a 0 or 1 is found --- 0 here
q2 x x R q2
q2 y y R q2
q2 X X R q2
q2 Y Y R q2  move right passing x,y,X,Y
q2 B X L q1  Have placed an X, move left

q1 1 y R q3  until a 0 or 1 is found --- 1 here
q3 x x R q3
q3 y y R q3
q3 X X R q3
q3 Y Y R q3  move right passing x,y,X,Y
q3 B Y L q1  Have placed an X, move left

q1 B B R q4  q1 sees a blank, all digits are changed
q4 x B R q4  erase x
q4 y B R q4  erase y
q4 X 0 R q4  change X to 0
q4 Y 1 R q4  change Y to 1
q4 B B L q5  all changed, move left
q5 0 0 L q5
q5 1 1 L q5
q5 B B R q6  and stop
```

```
tm reverse.tu sample-bitstrings
input: 1101
output:     1011

input: 0
output:  0

input: 00
output:   00

input: 01
output:   10

input:
output:

input: 1111
output:     1111

input: 1100
output:     0011

input: 0100
output:     0010

input: 1110000110111
output:               1110110000111
```

Question 5. Prove that if u, v, x, y are bitstrings and u, v are valid encodings of Turing machines and $ux = vy$, then $u = v$ and $x = y$.

Answer

The empty machine is encoded as 1111. All other machines are encoded as $1110 \ldots 0111$ and the three 1s occur only at the beginning and end. If $ux = vy$, then either $u = x = 1111$ or, because 111 occurs exactly twice in each, u and v have the same length and coincide. In either case, $u = v$, and therefore $x = y$. ∎