# MAU11602 fifth quiz, week 10, Wed 30/3/22 due on Blackboard, 12 noon, Wed 13/4/22

***This is the last assignment. There will be no Turing program.***

**Rules and procedures.**

**1.** Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

**Question 1.** Let $A = \{m : \ \phi_m(n) \downarrow 0\}$ (for all $n$), $B = \{m : \ \phi_m(n) \downarrow 1\}$. Prove that $A$ and $B$ are recursively inseparable. (Using the fixed point theorem).

**Question 2.** Rice's Theorem is about recursively enumerable sets rather than partial recursive functions. Recursively enumerable sets are domains of partial recursive functions.

Let $R$ be the set of recursively enumerable sets, so for each $S \in R$, there exists an $m$ such that $S = \{n : \ \phi_m(n) \downarrow\}$. A property $P$ of r.e. sets, i.e., a subset of $R$, is *trivial* if $P = \emptyset$ or $P = R$. Let $P$ be a nontrivial property. Prove that $P$ is not recursive…or, rather that.

$$(*) \quad \{m : \ \{n : \ \phi_m(n) \downarrow\} \in P\}$$

is not recursive, using the Fixed Point Theorem.

This is Rice's Theorem: no nontrivial property of recursively enumerable sets is recursive.

**Question 3.** Answer this question, without too much formality: is the set

$$\{m : m \text{ encodes a TM which has } > 25 \text{ quintuples}\}$$

recursive?

**Question 4.** Granted, with suitable encodings, that the set of provable formulae in PA is recursively enumerable but not recursive, is the set of unprovable formulae in PA recursively enumerable? Give reasons.

**Question 5.** Let $K$ be the first-order theory with multiplication $xy$, identity 1, equality axioms, and other proper axioms

$$\text{(i) } x_1(x_2x_3) = (x_1x_2)x_3 \quad \text{and} \quad \text{(ii) } \forall x_1 \exists x_2 (x_1x_2 = 1)$$

Let $K'$ be the theory like $K$, with a new unary function $f$, and with (ii) replaced by the axiom

$$\text{(ii')} \ \forall x_1 (x_1 f(x_1) = 1)$$

Given that $K$ is consistent, use the Completeness Theorem to show that $K'$ is a consistent extension of $K$.