MAU11602 fourth quiz, week 8, Wed 16/3/22
due on Blackboard, 12 noon, Wed 30/3/22

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. The following set of clauses, from Quiz 2, is inconsistent.

$$\overline{U}VY,\ \ UV\overline{Y},\ \ U\overline{V}Y,\ \ \overline{U}\,\overline{V}\,\overline{Y},\ \ UX,\ \ \overline{U}\,\overline{X},\ \ VWX,\ \ \overline{V}W\overline{X},$$
$$V\overline{W}\,\overline{X},\ \ \overline{V}\,\overline{W}X,\ \ WY,\ \ \overline{W}\,\overline{Y}$$

Every truth-assignment can be represented succinctly as a bitstring, taking the variables $UVWXY$ in that order. For example, the string 10110 means

$$U \mapsto 1, V \mapsto 0, W \mapsto 1, X \mapsto 1, Y \mapsto 0$$

Every truth assignment contradicts at least one clause in the above list. Give one clause contradicting each of the truth-assignments given below.

(i) 01100, (ii) 11011, (iii) 01110, (iv) 10001, (v) 00101

Question 2. Give a first-order theory K with formulae A and B and an interpretation I such that

$$A \vdash_K B$$

but $A \implies B$ is false in I.

Question 3. The 'numerals' $\overline{n}$ are important Peano-arithmetic terms defined recursively as follows

$$\overline{0} = 0; \quad \overline{n+1} = s(\overline{n}).$$

So $\overline{3} = s(s(s(0)))$.

Prove (in PA, Peano Arithmetic) that $\overline{2} + \overline{3} = \overline{5}$.

Equality reasoning: if $u = v$ is an axiom or has been proved, you may assume $v = u$ automatically (symmetry) and also assume automatically $u = v, v = w \vdash u = w$.

Question 4. Prove 19.9: $x + y = y + x$ by induction on y, using any result up to 19.8. (19.7 may help).

Question 5. Define $x \leq y$ to mean $\exists z(x + z = y)$ and $x < y$ to mean $(\exists z \neq 0)(x + z = y)$.

Prove: $x < y \implies s(x) \leq y$.