MAU11602 third quiz, week 6, Wed 2/2/22 due on Blackboard, 12 noon, Wed 16/3/22

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. $(\exists x_i A)$ is an abbreviation for $(\neg(\forall x_i(\neg A)))$. Give a direct definition of

$$I, \sigma \models \exists x_i A$$

Question 2. This question aims to show that Theorem 13.1 is false unless the 't free for x_i' condition is met.

Let $A(x_1)$ be $\exists x_2(x_1 \neq x_2)$ and $t = x_2$, so $A(t)$ is $\exists x_2(x_2 \neq x_2)$.

Let I be an interpretation with domain $\mathbb{N}$ with $=^I$ the equality relation on $\mathbb{N}$.

Let $\sigma = 1, 2, 3, 4 \ldots$ so $\sigma_4 = 4$ and so on.

According to Theorem 13.1, σ' is $\sigma_{1 \mapsto 2}$, since $\tau^\sigma = \sigma_2 = 2$.

Show that one of $A^{\sigma'}, A(t)^\sigma$ is true and the other false, so Theorem 13.1 does not hold.

Question 3. Let K be the FOT with one function $+$ and two predicates $=, \leq$. Let I_1 be the interpretation over $\mathbb{N}$, with $=$ interpreted in the usual way, but $\leq$ interpreted indirectly.

$$m \leq^{I_1} n \iff \text{for some } k \text{ in } \mathbb{N},\ m = n + k.$$

Let I_2 be the interpretation like I_1, except that

$$m \leq^{I_2} n \iff \text{for some } k \text{ in } \mathbb{N},\ n = m + k.$$

Verify whether the following is true in either interpretation:

$$\exists x_1 \forall x_2 (x_1 \leq x_2)$$

Question 4. Suppose K is the usual axiom system for group theory, with the formal language $1, f_1, f_2, P_1$ (identity, product, inverse, equality). Briefly, the axioms are associativity, left and right identity, left and right inverse.

We include a third binary function, f_3. We represent it informally as $x_1 \circ x_2$.

$$x_1 \circ x_2 = (x_1 x_2)(x_1^{-1} x_2^{-1})$$

Convert the following, conventional, formulae into the formal language of K:
(i) $x_1 x_2 = x_2 x_1$
(ii) $(x_1 x_2)^{-1} = x_1^{-1} x_2^{-1}$
(iii) $\circ$ is associative.

Let D be the collection of all 2×2 invertible real matrices, and I the interpretation with domain D, where f_1^I is matrix product, 1^I is the identity matrix, and f_2^I is matrix inverse. Let σ be the following snapshot.

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \ \sigma_2 = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, \ \sigma_3 = \begin{bmatrix} 17 & 20 \\ -12 & -14 \end{bmatrix}, \ \sigma_4 = \begin{bmatrix} 18 & 20 \\ -12 & -13 \end{bmatrix},$$
$$\sigma_5 = \begin{bmatrix} -7 & -10 \\ 6 & 8.5 \end{bmatrix}, \ \sigma_6 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ \sigma_7 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sigma_8 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \sigma_9 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Values of σ_i for $i \geq 10$ do not affect the calculations.

(iv) Say if the formula $x_3 x_4 = x_4 x_3$ is true under σ.

(v) Say if the formula $x_1 x_6 = x_6 x_1$ is true under σ.

Question 5. And check the following for truth under the snapshot σ:

(i) $x_1 x_5 = x_7$

(ii) $x_5 x_3 = x_7$

(iii) $x_8 \circ (x_9 \circ x_9) = (x_8 \circ x_9) \circ x_9$.

Hint. Invertible matrices A, B commute if and only if $A \circ B = I$ (the identity matrix).