26 Syllabus for 2022 exam

There will be a 3-hour take-home exam. 4 questions; credit will be given to the best 3 answers.

A question may cover different topics from the list below.

You should go back over the class notes, and the quizzes, to prepare for the exam. You will not be asked to prove the big results, such as the completeness of resolution or SC or first-order theories. The only first-order theory covered in any depth is Peano Arithmetic.

- Turing machines: construct machines for simple tasks. Encoding as bitstrings. The Halting Problem. Universal Turing machine. Recursive and recursively enumerable sets. Recursive and partial recursive functions.

- Truth-functions, DNF and CNF, realising a truth-function as a CNF or DNF, tautologies, inconsistent formulae, resolution, justification of resolution, completeness of resolution.

- Sentential Calculus (SC), axioms and MP. Justification of MP. Deduction Theorem. Completeness of SC.

- First-order languages, terms and formulae. Free and bound occurrences of variables in a formula. Idea of 't is free for x_i in $A(x_i)$.' First-order theories.

  Interpretations, snapshots. Truth in an interpretation, and models. Examples of interpretations. Justification of Axiom schemes IV and V, and Generalisation.

- Deduction theorem and its limitations in first-order logic; when it can be invalid. The 'fix' rule and its limitations; when it can be invalid.

- Complete and consistent theories. Complete consistent extensions. Scapegoat theories. Interpretations and models of first-order theories. Completeness of first-order logic.

- Equality theories, $\exists^*$ and $\exists!$.

- Peano arithmetic, and its development up to the 'division algorithm.'

- Primitive recursive functions. The 'numerals' $\overline{0}, \overline{1}, \ldots$

  Primitive recursive functions are representable in Peano Arithmetic.

- Length-lex encoding of bitstrings. Concatenation of bitstrings, with length-lex encoding, is primitive recursive.

- Encoding a TM M and a halting computation of M as a single number; the relation $H(m, n, r, s)$, $A(\overline{m}, \overline{n}, \overline{r}, \overline{s})$.

- The partial recursive functions $\phi_m(n)$. Recursive inseparability and the Fixed Point Theorem.

- Theorems of Peano Arithmetic and truth in $\mathbb{N}$, recursive inseparability. Four important theorems: (1) provability in PA is undecidable, (2) truth in $\mathbb{N}$ is undecidable (Tarski), (3) PA is incomplete (Gödel-Rosser), and (4) provability in a predicate calculus is (often) undecidable.

- Integers can be infinite.