MAU11602 second quiz, week 4, Wed 16/2/22 ANSWERS

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard.

Question 1. Give a CNF with the following truth-table.

W	X	Y	Z	
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1

W	X	Y	Z	
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Answer.

$$W \lor X \lor \overline{Y} \lor \overline{Z}, \quad W \lor \overline{X} \lor \overline{Y} \lor Z, \quad \overline{W} \lor X \lor Y \lor \overline{Z}, \quad \overline{W} \lor \overline{X} \lor \overline{Y} \lor Z, \quad \overline{W} \lor \overline{X} \lor \overline{Y} \lor \overline{Z}$$

Question 2. Show using resolution that the following CNF is inconsistent

$$\overline{U}VY, \quad UV\overline{Y}, \quad U\overline{V}Y, \quad \overline{U}\,\overline{V}\,\overline{Y}, \quad UX, \quad \overline{U}\,\overline{X}, \quad VWX, \quad \overline{V}W\overline{X},$$
$$V\overline{W}\,\overline{X}, \quad \overline{V}\,\overline{W}X, \quad WY, \quad \overline{W}\,\overline{Y}$$

Answer.

$$\overline{U}VY, \overline{W}\,\overline{Y} \mapsto \overline{U}V\overline{W} \quad UV\overline{Y}, WY \mapsto UVW \quad U\overline{V}Y, \overline{W}\,\overline{Y} \mapsto U\overline{V}\,\overline{W} \quad \overline{U}\,\overline{V}\,\overline{Y}, WY \mapsto \overline{U}\,\overline{V}W$$
$$UX, \overline{V}W\overline{X} \mapsto U\overline{V}W \quad UX, \overline{W}\,\overline{X} \mapsto UV\overline{W} \quad \overline{U}\,\overline{X}, VWX \mapsto \overline{U}VW \quad \overline{U}\,\overline{X}, \overline{V}\,\overline{W}X \mapsto \overline{U}\,\overline{V}\,\overline{W}$$
$$UVW, UV\overline{W} \mapsto UV \quad U\overline{V}\,\overline{W}, U\overline{V}W \mapsto U\overline{V} \quad \overline{U}V\overline{W}, \overline{U}VW \mapsto \overline{U}V \quad \overline{U}\,\overline{V}W, \overline{U}\,\overline{V}\,\overline{W} \mapsto \overline{U}\,\overline{V}$$
$$U\overline{V}, U\overline{V} \mapsto U \quad \overline{U}V, \overline{U}\,\overline{V} \mapsto \overline{U} \quad U, \overline{U} \mapsto \square$$

Question 3. Let A and B be formulae of SC with the property that every interpretation (truth assignment) which satisfies A also satisfies B. Prove

$$A \vdash_{\mathrm{SC}} B$$

Answer.

$A \implies B$ is a tautology, so it is provable in SC: $\vdash_{\mathrm{SC}} A \Rightarrow B$. Using MP, $A \vdash_{\mathrm{SC}} B$. ∎

Question 4. Prove
$$\vdash_{\mathrm{SC}} A \wedge (B \wedge C) \implies (A \wedge B) \wedge C.$$
You may use the notes up to Lemma 10.17 (vi), but nothing after that. (Hint: you needn't look too far back.)

Answer. 1. $A \wedge (B \wedge C)$ (hyp).
2. A (1, Lemma 10.17, (iv)).
3. $B \wedge C$ (1, L, (v)).
4. B (3, L, (iv)).
5. C (3, L, (v))
6. $(A \wedge B)$ (2,4, L, (vi))
7. $(A \wedge B) \wedge C)$ (6,5,L (vi)
so 7 can be derived from (1), therefore using the Deduction Theorem, $A \wedge (B \wedge C) \implies (A \wedge B) \wedge C$. ∎

Question 5. Prove
$$\vdash_{\mathrm{SC}} (A \vee (B \wedge C)) \implies ((A \vee B) \wedge (A \vee C)) \qquad (*)$$
(This is part of (viii) in Lemma 10.17. Hint: $(\neg A) \implies (B \wedge C) \ldots$)

Answer.
1. $(\neg A) \implies (B \wedge C))$ (Hypothesis)
2. $\neg A$ (Hypothesis)
3. $(B \wedge C)$ (1,2,MP)
4. B (3, Lemma 10.17, (iv))
5. C (3, Lemma, (v))
6. $(\neg A) \Rightarrow B$ (DT with (2) as hypothesis.)
7. $(\neg A) \Rightarrow C$ (same again)
8. $((\neg A) \Rightarrow B) \wedge ((\neg A) \Rightarrow C)$ (6,7, Lemma, (vi))
9. (1) $\implies$ (8) by the deduction theorem, and that is the desired formula $(*)$. ∎