

$7/13$ again . $14/13$ for first digit.

Repeat quotient remainder double

$$\begin{array}{cccccccccccc} \frac{14}{13} & 2\frac{1}{13} & \frac{4}{13} & \frac{8}{13} & \frac{16}{13} & \frac{6}{13} & \frac{12}{13} & \frac{24}{13} & \frac{22}{13} & \frac{18}{13} & \frac{10}{13} & \frac{20}{13} \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ \frac{1}{13} & 2\frac{1}{13} & \frac{4}{13} & \frac{8}{13} & 3\frac{1}{13} & \frac{6}{13} & \frac{12}{13} & \frac{11}{13} & \frac{9}{13} & \frac{5}{13} & \frac{10}{13} & \frac{7}{13} \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} 14/13 \dots$$

.100010011101 recurring

Yes, there was an error last Friday at $\frac{12}{13}$

Single precision "float"



Exponent BIASED so $b - 127 = e$

sign = 1 negative

The floating point rep. of a number $x \neq 0$
given its true value is $\pm 2^e 1.a_1 \dots a_{24} \dots$

is (a) sign (b) biased exponent (add 127)

(c) mantissa $a_1 \dots a_{23}$ (rounded up if $a_{24} = 1$)

(d) 'little endian'

C programming, Monday 9/11/20

A floating-point number is an encoding of a number of the following kind:

(sign) $M \times 2^e$, where $1 \leq M < 2$

This is related to 'scientific notation' which you can produce in C with %e format:

```
printf("%e\n", 12345.678);  
output
```

1.234568e+04

meaning

1.234568×10^4

single precision

1 sign bit 8 exponent 23
mantissa

double

1 sign bit 11 exponent 52
mantissa

Example Convert $(-5.0)/104$ to single precision

the sign is negative. Sign bit is 1 for negative.

The exact meaning of a 32-bit float,
partitioned as follows

(sign 1 bit) ('biased exponent' b) ('mantissa'
 $a_1 a_2 \dots a_{23}$)

is

(sign) $(2^{b-127}) (1.a_1a_2 \dots a_{23})$

except for zero where everything is zero.

Working with (-5.0)/104

We need first to multiply 5.0/104 by a power of 2 to bring it into the correct range for a mantissa.

That is, at least 1 and less than 2.

Well, if you multiply by 32 you get 20.0/13 which is in the correct range. That is 2^5 .

Therefore

$M \times 2^5 = 5.0/13$. The correct exponent is -5.

Add it to 127 to get the biased exponent, 122.

Convert this to an 8-bit binary number (face value)

Or more quickly convert to hex. $122 = 16^*7 + 10$
7a in hex or 0111 1010 in binary. This is the biased exponent.

Example $x : \frac{-5}{104} = -\frac{5}{13 \times 8}$ SIGN 1 (a)

Multiply $\frac{5}{104}$ by 2^5 : $\frac{20}{13}$ between 1 and 2.

So exponent $e = -5$ biased 127-5

or in binary

$$\begin{array}{r} 01111111 \\ -101 \\ \hline 01111010 \end{array} \text{BIASED EXP} \quad (b)$$

mantissa. $\frac{20}{13} = 1 + \frac{7}{13}$ DONE

$a_1 \dots a_{24} = 100010011101100010011101$ round up
 mantissa 10001001110110001001111

altogether

$$\begin{array}{ccccccccc} 1 & 011 & 1101 & 0100 & 0100 & 1110 & 1100 & 0100 & 1111 \\ b & d & 4 & 4 & e & c & 4 & f & \end{array} \quad 4f \text{ ec } 44 \text{ bd}$$

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
1	0	0	0	1	0	0	1	1	1	0	1

So $20.0/13 =$

1. 1000 1001 1101

----- recurring

Check:

$$1 + \left(\frac{2205}{4096} \right) \left(\frac{1}{4096} + \frac{1}{4096^2} \dots \right)$$

$$= 1 + \frac{2205}{4095} = 1 + 7/13 = 20/13.$$

Correct (sum geometric series)

24 bits of this infinite sequence

1000 1001 1101 1000 1001 1101

The 24th bit is 1: ROUND UP.

Equivalently, add 1 to 23 bits

1000 1001 1101 1000 1001 110

+

1

1000 1001 1101 1000 1001 111

sign biased exp

mantissa

1 0111 1010 1000 1001 1101 1000 1001 111

1011 1101 0100 0100 1110 1100 0100 1111

b d 4 4 e c 4 f

One more complication

4f ec 44 bd little endian

Arrays and initialisation

```
int a[100]; double b[200];
```

a[0], a[1], ..., a[99] A PECULIARITY of C

Distinguish

```
int a[10]; --- 10 is the size of the array  
printf("%d\n", a[5]) --- a[5] is the sixth  
element of the array
```

Reading in an array from the keyboard
Look at section 8.1 of the web notes

```
#include <stdio.h>
main()
{ double a[1000]; // big enough?
  int count; double x; count = 0;
  while ( scanf("%lf", &x) == 1 )
  { if ( count < 1000 ) // Note!!!!
    { a[count] = x; count = count+1; }
  }
  printf(" %d numbers read\n", count);
```

```
int i;
for (i=0; i<count; i=i+1)
{ printf(" %f", a[i]); }
printf("\n");
printf(" and in reverse order\n");
for (i=count-1; i>=0; i=i-1)
{ printf(" %f", a[i]); }
} // end of program
```

Initialisation

```
int x = 4; double y = 2.3;
```

(i) saves keystrokes

(ii) Risky because one might assume
 $x == 4$ after it was changed

(iii) Very useful with arrays, especially for
'tables' which don't change.

(iv) a character string is an array of characters

```
char hello[6] = "hello";
```

```
char *monthname[12] = {"January", "February", "March", ...};
```

ONLY INITIALISATION

NO BULK ASSIGNMENTS

Initialising arrays, very convenient in C.

```
int month_length[12] =  
{31,28,31,30,31,30,31,31,30,31,30,31};
```

Note (i) curly braces (ii) followed by a semicolon.
Another inconsistency in C.

Characters: `char' is used for declaring
character data, actually single bytes (groups of
8 bits)

```
char x; char y = 'z'; char newline = '\n'
```

Character strings: arrays of characters

```
char hello[6];
```

A character string is usually assumed to be formed of ASCII characters. Initialisation is possible:

char hello[6] = "hello"; NOTE 6 not 5

but assignment is NOT possible
hello = "hello"; is invalid

You cannot assign values directly to an array.
On the other hand
hello[1] = 'a'
is valid, and it changes the array to
"hallo".

Why do you need 6 characters to hold a character string of length 5?

char hello[5] = {'h','e','l','l','o'};

is valid C code but is missing something vital:
a marker for the end of the string. For that
the null character '\0' is used (it does not
represent any printable character).

char hello[6] = "hello";

and

char hello[6] = {'h','e','l','l','o','\0'}

are both correct and equivalent.

Useful example: char
hex_digit[17] = "0123456789abcdef";

Day in week calculation, for a day in this century.

dd mm yy

$1 \leq mm \leq 12, 0 \leq yy \leq 99$

add together

6

yy

yy/4

Total number of days in the months before the mm-th month

+ 6 if $mm < 3$ and $yy \% 4 = 0$

And take the remainder modulo 7. This will give an integer from 0 to 6, with 0 for Sunday.