Maths 3468 quiz answers

3: Friday 9/3/12

Consider computation of two bilinear forms R, I as shown. Let a, b, c, d be the initial values of A, B, C, D, and (slightly confusing) write $\vec{a}$ for a, b and $\vec{b}$ for c, d.

$$\begin{aligned} X &\leftarrow AC \\ Y &\leftarrow BD \\ Z_1 &\leftarrow A+B \\ Z_2 &\leftarrow C+D \\ Z &\leftarrow Z_1 Z_2 \\ I_1 &\leftarrow Z - X \\ R &\leftarrow X - Y \\ I &\leftarrow I_1 - Y \end{aligned}$$

(1) What is $R(\vec{a}, \vec{b})$? **Answer.** $ac - bd$

(2) What is $I(\vec{a}, \vec{b})$? **Answer.** $ad + bc$.

(3) What is the point of this calculation?
Answer. Multiplying two complex numbers with 3 real multiplications.

(4)

$$\sum_i \left| a_i \frac{\partial R}{\partial a_i} \right| + \sum_j \left| b_j \frac{\partial R}{\partial b_j} \right| = 2|ac| + 2|bd|$$

(5) Ditto, with I in place of R. **Answer.** $2|ad| + 2|bc|$.

(6) For $V = X, Z_1, Z$, compute $\Delta R_V(\vec{a}, \vec{b})$.

Answer This requires R to be expressed with V treated as an input. So $R = X - bd$ (independent of Z_1, Z).

$$\begin{aligned} \Delta R_X &= X = ac \\ \Delta R_{Z_1} &= \Delta R_Z = 0 \end{aligned}$$

(7) For $V = X, Z_1, Z$, compute $\Delta I_V(\vec{a}, \vec{b})$.
Answer. $I = Z - X - Y = Z_1 Z_2 - X - Y$.

$$\begin{aligned} \Delta I_X &= -X = -ac \\ \Delta I_{Z_1} &= Z_1 Z_2 = (a+b)(c+d) \\ \Delta I_Z &= Z = (a+b)(c+d) \end{aligned}$$

(8) Determine whether the program is (simultaneously, i.e., for both R and I) Brent stable, restricted Brent stable, or strongly stable.

Answer. Both are permutation forms, so restricted Brent stability is equivalent to strong stability.

Some but not all of the ΔT_V have been computed.

$$\Delta R_X = ac \quad \Delta R_Y = -bd \quad \Delta R_I = \Delta R_{I_1} = 0$$

$$|(\vec{a}, \vec{b})|_R = \max(|ac|, |bd|)$$

Computation of R is restricted Brent stable and hence strongly stable and also Brent stable.

Concerning I,

$$\Delta I_X = -ac \quad \Delta I_Y = -bd$$
$$\Delta I_{Z_1} = \Delta I_Z = (a+b)(c+d)$$
$$\Delta I_{I_1} = K_1 = Z - X = (a+b)(c+d) - ac = ad + bc + bd$$

Only the last two formulae need looking ad. But $|(a+b)(c+d)| \leq (2\|\vec{a}\|)(2\|\vec{b}\|)$, similarly for ΔI_{I_1}; the program is Brent stable.

$$|(\vec{a}, \vec{b})|_I = \max(|ad|, |bc)$$

Let $\vec{a} = 0, 1$ and $\vec{b} = 1, 0$. then $\Delta I_Z = 1$ but $|(\vec{a}, \vec{b})|_I = 0$, so it is not restricted Brent stable.