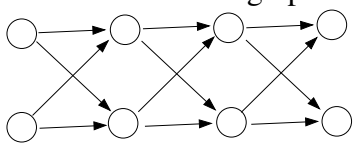
Maths 3468 quizzes

1: Friday 27/1/12

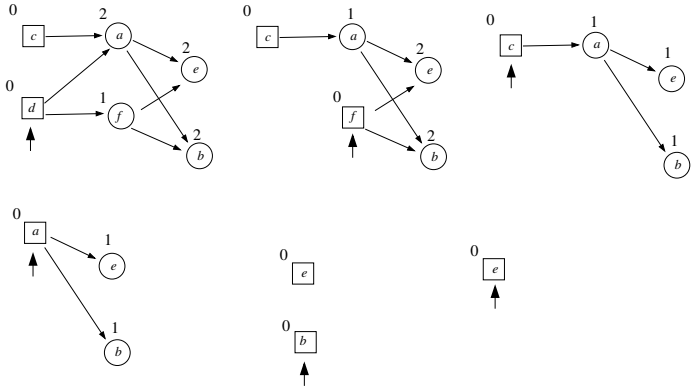
(1) How many topological orders are there on the digraph shown below?



If we call this a 4-rung ladder, how many topological orders are there on a k-rung ladder? (No proof needed.)

Answers. 16 and 2^k.

(2) Apply the topological sort algorithm (first kind, not DFS) to the graph below.



(3) Recall that if DFS is applied to a digraph, then (the nodes in) each SCC span a prefix subtree in the DFSF.

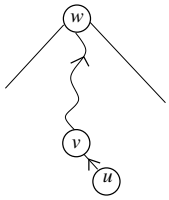
Suppose that DFS is applied and u and v are nodes in different SCCs, and v is a descendant of u. Show that the scc-root of v is also a descendant of u. (Hint: it's either a descendant or an ancestor).

Proof. v is a descendant of u and v is a descendant of w (the SCC root of v). Either w is a descendant of u or an ancestor. It cannot be an ancestor, since all nodes on the tree branch from w to v are in the same SCC. Therefore it is a descendant. ∎

Questions 3,4, and 5, are related to Sharir's SCC method, which is not covered in lectures. The method is: Phase 1. DFS G. Phase 2. Let G' be G with all its edges reversed.

Note that G' and G have the same equivalence classes under strong connectivity.

Phase 2. DFS G', always restarting the DFS at the node with highest Phase 1 postorder rank. Then the trees in the phase 2 DFSF span the SCCs of G.



The above figure illustrates a phase-2 DFS tree (hence the indicated direction of tree edges). Assume that w is an SCC root from Phase 1, call it K, and $v \in K$ (hence all nodes on the path from w to v are in K). Note (u, v) is an edge of the original digraph G.

(4) Prove that if u precedes v in (phase 1) preorder, then u follows v and w in phase 1 postorder. (Use (3)).

Proof. If u precedes v in preorder, then v becomes a descendant of u, and so is w (q 3), and they precede u in postorder. ∎

(5) Prove that if u follows v (hence also w) in (phase 1) preorder, and u precedes w in postorder, then $u \in K$.

Proof. Under these assumptions u is a descendant of w in phase 1 DFS. There is a path from v to w, hence there is a path from v to u: hence u, v are in the same SCC. ∎