UNIVERSITY OF DUBLIN

XMA1E21

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF BAI/MEMS/MSISS **Trinity Term 2007**

Mathematics 1E2

Wednesday, 23 May, 2007 RDS 2–5 pm

Dr. Ó Dúnlaing

Attempt ALL questions.

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. Use Gauss-Jordan elimination, no other method, to find all solutions to the system

$$\begin{bmatrix} 1 & -3 & -5 & -5 \\ -3 & 10 & 17 & 17 \\ 3 & -10 & -17 & -16 \\ 2 & -5 & -8 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -9 \\ 32 \\ -30 \\ -11 \end{bmatrix}$$

2. (a) Evaluate the determinant

$$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 2 & 6 & 6 & 2 \\ 1 & 4 & 5 & 4 \\ 1 & 1 & 4 & 0 \end{vmatrix}$$

by bringing to upper triangular form.

(b) Evaluate the same determinant by cofactor expansion along the first row.

3. (a) Find eigenvalues and eigenvectors (an orthonormal system) for the symmetric matrix

$$A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

(b) Hence solve the simultaneous differential equations

$$\frac{dx}{dt} = 6x + 2y; \quad \frac{dy}{dt} = 2x + 3y,$$

including an explicit formula for e^{At}. The matrix A is the same as in the first part of this question.

4. In all parts of this question, A is an $m \times m$ matrix and A' is a RREF (reduced row-echelon form) of A (it is unique). You may assume that there exists an invertible $m \times m$ matrix P such that $A' = PA$.

(a) Define: 'A is invertible.'

(b) Suppose that A is invertible and B is another $m \times m$ matrix. Prove that AB is invertible if and only if B is invertible. You may assume without proof A^{-1} is invertible.

(c) Prove that A is invertible if and only if $A' = I$.

5. (a) A jar contains red and white balls; exactly $2/3$ are red and $1/3$ are white.

The following experiment is performed four times: the jar is shaken thoroughly, a ball is taken out, its colour noted, and the ball returned to the jar.

Thus, each time a ball is taken out, the probability that it is red is $2/3$.

For $0 \le i \le 4$, calculate the probability, as a fraction, that exactly i times (out of 4) the colour was red.

(b) Let E be the event in which

an odd number of times, the colour was red.

For $0 \le i \le 4$, calculate the probability that

red occurred i times, given E.

(c) Three assembly lines A,B,C produce the same component; B and C at the same rate, A at twice that rate. The probability of A,B,C, respectively, producing a defective component, is $4\%, 5\%, 7\%$, respectively.

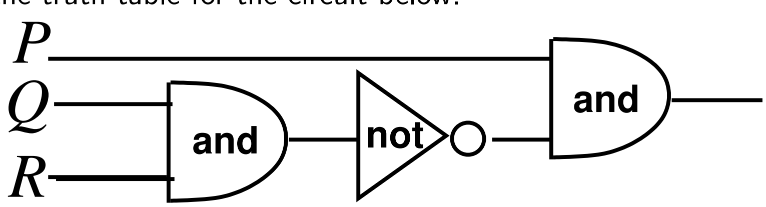
Calculate the probabilities that a given defective component has been produced on A,B,C, respectively.

6. (a) Prove by resolution that the following set of clauses is inconsistent.

$$\overline{A},\ ABC,\ A\overline{B}\,\overline{C},\ B\overline{C},\ \overline{B}C$$

It represents the formula $\overline{A} \wedge (A \vee B \vee C) \wedge (A \vee \overline{B} \vee \overline{C}) \wedge (B \vee \overline{C}) \wedge (\overline{B} \vee C)$.

(b) Give the truth-table for the circuit below.



(c) Construct a DNF (disjunctive normal form formula, such as $(X \wedge \overline{Y} \wedge Z) \vee (\overline{X} \wedge Y \wedge Z)$), for the circuit (and the truth-table).

7. Find the general solutions to the following differential equations and recurrences.

(a) $dy/dx = -2xy^2$

(b) $dy/dx - 2xy = e^{x^2}$

(c) $y_{n+1} - 2y_n = 1$

(d) $y_{n+1} - (n+1)y_n = (n+1)!$

8. Find the general solutions to the following differential equations and recurrences.

(a) $(D^2 - 3D + 2)y = e^{3x}$

(b) $(D^2 - 3D + 2)y = e^x$

(c) $(E^2 - 3E + 2)y_n = 2^n$

© UNIVERSITY OF DUBLIN 2007