SECTION NO. (1-7)__________

Name:____________________

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Mathematics 1E2 2006–07

HW 20 Due 30/4/07

This is the last homework. The year-end exam is on Wednesday 23 May from 2–5 pm at the RDS (Royal Dublin Society headquarters, Ballsbridge). You are to answer 8 questions out of 8. There is no choice. The exam generally looks like last year's, though there won't be quite so many parts to every question. You can use a non-programmable calculator, but you must write the make and model of your calculator on each answer book used.

Find *all* solutions to the following recurrences.

(1) $(E^2 - 7E + 12)y_n = 2^n$.

$$\begin{aligned}
(E-3)(E-4)y_n &= 2^n \\
(E-1)3^{-n}(E-4)y_n &= \frac{1}{3}(2/3)^n \\
3^{-n}(E-4)y_n &= 1-(2/3)^n \\
&\text{ignore } 1 \\
4^{n+1}(E-1)4^{-n}y_n &= -2^n \\
(E-1)4^{-n}y_n &= -\frac{1}{4}(1/2)^n \\
4^{-n}y_n &= -\frac{1}{2}(1-(1/2)^n) \\
&\text{ignore the } 1 \\
y_n &= 2^{n-1} \text{ particular} \\
y_n &= 2^{n-1} + A3^n + B4^n \text{ general}
\end{aligned}$$

(2) $(E^2 - 7E + 12)y_n = 3^n$.

$$\begin{aligned}
(E-4)(E-3)y_n &= 3^n \\
(E-1)4^{-n}(E-3)y_n &= \frac{1}{4}(3/4)^n \\
4^{-n}(E-3)y_n &= 1-(3/4)^n \\
&\text{ignore } 1 \\
3^{n+1}(E-1)3^{-n}y_n &= -3^n \\
(E-1)3^{-n}y_n &= -\frac{1}{3} \\
3^{-n}y_n &= -\frac{n}{3} \\
y_n &= -n3^{n-1} \text{particular} \\
y_n &= -n3^{n-1} + A3^n + B4^n \text{ general}
\end{aligned}$$

(3) $(E^2 - 4E + 4)y_n = 3^n$

$$\begin{aligned}
(E-2)^2 y_n &= 3^n \\
(E-1)2^{-n}(E-2)y_n &= \frac{1}{2}(3/2)^n \\
2^{-n}(E-2)y_n &= (3/2)^n - 1 \\
\text{ignoring the } -1 \\
(E-2)y_n &= 3^n \\
\text{same again} \\
y_n &= 3^n \text{ particular} \\
y_n &= 3^n + A2^n + Bn2^n \text{ general}
\end{aligned}$$

(4) $(E^2 - 2E + 2)y_n = 1$ (General real solution! You can write the real solution with the aid of cos and sin: $(1 \pm i) = \sqrt{2}e^{\pm i\pi/4}$.)

Let $\alpha = 1 + i$ and $\beta = 1 - i$. Using a formula from the notes,

$$\begin{aligned}
(E-\alpha)(E-\beta)y_n &= 1 \\
(E-\beta)y_n &= \frac{1}{1-\alpha} \\
y_n = \frac{1}{(1-\alpha)(1-\beta)} = \frac{1}{(-i)(i)} &= 1 \text{ particular (real) solution} \\
y_n &= A\alpha^n + B\beta^n + 1 \text{ general complex} \\
y_n &= 1 + (\sqrt{2})^n(A\cos(n\pi/4) + B\sin(n\pi/4))
\end{aligned}$$