SECTION NO. (1-7)

Name:____

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Mathematics 1E2 2006–07

HW 18 Due 16/4/07

(1) Find (real) eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$

$$(\lambda - 2)^2 - 9 = 0; \quad \lambda = 5, -1$$

$$5 : \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}; \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad -1 : \begin{bmatrix} -3 & -3 \\ -3 & -3 \end{bmatrix}; \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(2) Solve (calculating e^{At} explicitly)

$$\frac{dx}{dt} = 2x + 3y; \quad \frac{dy}{dt} = 3x + 2y$$

$$e^{At} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{e^{5t}+e^{-t}}{2} & \frac{e^{5t}-e^{-t}}{2} \\ \frac{e^{5t}-e^{-t}}{2} & \frac{e^{5t}+e^{-t}}{2} \end{bmatrix}$$

(3) Find (complex) eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$$\lambda^2 + 9 = 0; \lambda = \pm 3i \quad 3i : \begin{bmatrix} 3i & -3 \\ 3 & 3i \end{bmatrix}; \begin{bmatrix} i \\ -1 \end{bmatrix} \qquad -3i : \begin{bmatrix} -3i & -3 \\ 3 & -3i \end{bmatrix}; \begin{bmatrix} i \\ 1 \end{bmatrix}$$

(4) Solve (calculating e^{At} explicitly, using complex formulae for trig functions)

$$\frac{dx}{dt} = 3y; \quad \frac{dy}{dt} = -3x$$

$$S = \begin{bmatrix} i & i \\ -1 & 1 \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} -i/2 & -1/2 \\ -i/2 & 1/2 \end{bmatrix}$$

$$Se^{A't}S^{-1} = \begin{bmatrix} i & i \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{3it} & 0 \\ 0 & e^{-3it} \end{bmatrix}\begin{bmatrix} -i/2 & -1/2 \\ -i/2 & 1/2 \end{bmatrix} =$$

$$\begin{bmatrix} \cos 3t & \sin 3t \\ -\sin 3t & \cos 3t \end{bmatrix}$$