SECTION NO. (1-7)
Name: ______________________
ID: ______________

Mathematics 1E2 2006–07
HW 17 Due TUESDAY 10/4/07

Monday is a Bank Holiday. Give full answers including 'constants of integration.' Where possible, give y as a function of x, rather than a curve relating y to x.

(1) (8 marks) Solve $dy/dx = x/y$.

$$y\frac{dy}{dx} = x; \quad \int y dy = \int x dx; \quad \frac{1}{2}y^2 = \frac{1}{2}x^2 + \text{const}; \quad y^2 - x^2 = C.$$

(2) (8 marks) Solve $dy/dx = 3x^2y^2$

$$y^{-2}dy = 3x^2dx; \quad -\frac{1}{y} = x^3 + \text{const}; \quad \frac{1}{y} = C - x^3; \quad y = \frac{1}{C - x^3}.$$

(3) (12 marks) Solve $dy/dx - 2xy = 2xe^{x^2}$ **(CHANGED)**

$$\text{Homogeneous } \frac{du}{dx} - 2xu = 0 \ldots u = e^{x^2}$$

$$\ldots e^{x^2}\frac{dv}{dx} = 2xe^{x^2} \ldots$$

$$v = x^2 + C$$

$$y = (x^2 + C)e^{x^2}$$

(4) (8 marks) Solve $y_{n+1} - y_n = \frac{1}{(n+1)(n+2)}$. Hint: the right-hand side is $\frac{1}{n+1} - \frac{1}{n+2}$.

$$y_n = C + \sum_{r=0}^{n-1}\left(\frac{1}{r+1} - \frac{1}{r+2}\right) = \text{const} + \frac{1}{1} - \frac{1}{n+1} = C - \frac{1}{n+1}.$$

(5) (14 marks) Using (4), solve $y_{n+1} - (n+1)y_n = \frac{n!}{n+2}$

$$\text{Homogeneous } u_{n+1} - (n+2)u_n = 0 \quad u_{n+1} = (n+1)u_n$$

$$u_n = C\prod_{r=0}^{n-1}(r+1) = Cn!; \quad \text{take } C = 1:$$

$$(n+1)!v_{n+1} - (n+1)!n = \frac{n!}{n+2}$$

$$v_{n+1} - v_n = \frac{1}{(n+1)(n+2)}$$

$$v_n = C - \frac{1}{n+1}$$

$$y_n = Cn! - \frac{n!}{n+1}$$