

Mathematics 1E2 2006–07
HW 4 Due 7/11/06

Name: _____

ID: _____

(1) (12 marks). Let $P = (2, 5)$, $Q = (3, 4)$. (i) Calculate the matrix for converting from 'oblique' coordinates (in the ordered basis (P, Q)) to cartesian coordinates. (ii) Calculate the matrix for converting from cartesian to P, Q -coordinates

$$(i) S = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}, \quad (ii) S^{-1} = \frac{1}{8-15} \begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-4}{7} & \frac{3}{7} \\ \frac{5}{7} & \frac{-2}{7} \end{bmatrix}.$$

(iii) Calculate the 'new' coordinates of $(1, 2)$. $(2/7, 1/7)$.

(iv) Calculate the cartesian coordinates of the point whose new coordinates are $(1, 2)$. $(8, 13)$.

(2) (13 marks). Calculate the (i) new-to-old and (ii) old-to-new coordinate change matrices for the new ordered basis $P = (1, -1, 1)$, $Q = (2, 2, -1)$, $R = (-5, -6, 2)$.

$$(i) S = \begin{bmatrix} 1 & 2 & -5 \\ -1 & 2 & -6 \\ 1 & -1 & 2 \end{bmatrix} \quad (ii) \text{ see below } S^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{-1}{5} & \frac{2}{5} \\ \frac{4}{5} & \frac{-7}{5} & \frac{-11}{5} \\ \frac{1}{5} & \frac{-3}{5} & \frac{-4}{5} \end{bmatrix}$$

$$\begin{array}{cccccc} 1 & 2 & -5 & 1 & 0 & 0 & =R1 \\ -1 & 2 & -6 & 0 & 1 & 0 & +1*R1 \\ 1 & -1 & 2 & 0 & 0 & 1 & -1*R1 \\ \\ 1 & 2 & -5 & 1 & 0 & 0 & -2*R2 \\ 0 & 4 & -11 & 1 & 1 & 0 & *(1/4) =R2 \\ 0 & -3 & 7 & -1 & 0 & 1 & +3*R2 \\ \\ 1 & 0 & 1/2 & 1/2 & -1/2 & 0 & -1/2*R3 \\ 0 & 1 & -11/4 & 1/4 & 1/4 & 0 & +11/4*R3 \\ 0 & 0 & -5/4 & -1/4 & 3/4 & 1 & *(-4/5) =R3 \\ \\ 1 & 0 & 0 & 2/5 & -1/5 & 2/5 & \\ 0 & 1 & 0 & 4/5 & -7/5 & -11/5 & \\ 0 & 0 & 1 & 1/5 & -3/5 & -4/5 & \text{in rref} \end{array}$$

(3) (12 marks). (i) Give a point on the line $x + 2y = 3$. $(1, 1)$.

(ii) Give a point P such that \vec{OP} is perpendicular to this line. $(1, 2)$.

(iii) Calculate the positive normal to the vector \vec{OP} where $P = (15, -13)$. $(13, 15)$.

(iv) Calculate the distance from $(2, 3, 1)$ to $(4, 1, 2)$. $\sqrt{(4-2)^2 + (1-3)^2 + (2-1)^2} = \sqrt{9} = 3$.

(4) (13 marks). Apply the Gram-Schmidt orthogonalisation procedure to the points $(1, 2, 3)$ and $(4, 5, 6)$. Later.