Mathematics 1E2 2006–07
HW 2 Due TUESDAY 24/10/06

Name:_____

ID:_____

(1) (18 marks) Let

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}.$$

$$\text{(i)}AB = \begin{bmatrix} 12 & 19 \\ 11 & 17 \end{bmatrix}, \quad \text{(ii)}BA = \begin{bmatrix} 18 & 7 \\ 29 & 11 \end{bmatrix}, \quad \text{(iii)}BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{(iv)}CB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{(v)}A(BC) = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}, \quad \text{(vi)}(AB)C = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}.$$

(2) (8 marks) Invert the matrix

$$\begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix}$$

(i) Using the $1/(ad-bc)\ldots$ formula

$$\frac{1}{7-10}\begin{bmatrix} 7 & -5 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-7}{3} & \frac{5}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

(ii) Using annotated GJE on the usual 2×4 matrix.

```
1    5    1    0   =R1          1    5    1    0   -5*R2
2    7    0    1   -2*R1        0   -3   -2    1  *(-1/3)  =R2

1    0 -7/3  5/3
0    1  2/3 -1/3  in rref
```

(3) (10 marks). Using annotated GJE on the usual 3×6 matrix, determine whether the following matrix is invertible, and calculate the inverse if it is. (There are some fractions in the calculation.)

$$\begin{bmatrix} 1 & 1 & 6 \\ 0 & 2 & 2 \\ 3 & 2 & 17 \end{bmatrix}$$

```
1  1  6  1  0  0 =R1          1  0  5  1 -1/2  0 -1*R3
0  2  2  0  1  0              0  1  1  0  1/2  0
3  2 17  0  0  1 -3*R1        0  0  0 -3  1/2  1  *(-1/3) =R3
                               ... NOT INVERTIBLE! Can stop here.

1  1  6  1  0  0 -1*R2        1  0  5  0 -1/3  1/3
0  2  2  0  1  0  *(1/2)=R2   0  1  1  0  1/2  0
0 -1 -1 -3  0  1 +1*R2        0  0  0  1 -1/6 -1/3  in rref
```

in the calculation.)

$$\begin{bmatrix} 1 & -3 & 1 & -6 \\ 1 & -2 & 1 & -5 \\ 3 & -9 & 3 & -19 \\ 0 & -2 & 1 & -2 \end{bmatrix}$$

```
1 -3  1 -6   1  0  0  0  =R1
1 -2  1 -5   0  1  0  0  -1*R1
3 -9  3 -19  0  0  1  0  -3*R1
0 -2  1 -2   0  0  0  1

1 -3  1 -6  1  0  0  0  +3*R2
0  1  0  1 -1  1  0  0  =R2
0  0  0 -1 -3  0  1  0
0 -2  1 -2  0  0  0  1  +2*R2

1  0  1 -3 -2  3  0  0
0  1  0  1 -1  1  0  0
0  0  0 -1 -3  0  1  0 swap
0  0  1  0 -2  2  0  1 swap
```

```
1  0  1 -3 -2  3  0  0  -1*R3
0  1  0  1 -1  1  0  0
0  0  1  0 -2  2  0  1  =R3
0  0  0 -1 -3  0  1  0

1  0  0 -3  0  1  0 -1  +3*R4
0  1  0  1 -1  1  0  0  -1*R4
0  0  1  0 -2  2  0  1
0  0  0 -1 -3  0  1  0 *(-1) =R4

1  0  0  0  9  1 -3 -1
0  1  0  0 -4  1  1  0
0  0  1  0 -2  2  0  1
0  0  0  1  3  0 -1  0 in rref
          <-inverse->
```