Mathematics 1E2 2006–07
Due 16/10/06

Name:_____

ID:_____

$$(1)\begin{bmatrix}0\\0\end{bmatrix}\ (2)\begin{bmatrix}0&1\\1&0\end{bmatrix}\ (3)\begin{bmatrix}0&1\\0&0\end{bmatrix}\ (4)\begin{bmatrix}1&0&0\\0&0&1\end{bmatrix}\ (5)\begin{bmatrix}1&0&1&0\\0&0&1&0\\0&0&0&0\end{bmatrix}\ (6)\begin{bmatrix}1&2&0&0&3\\0&0&0&1&0\\0&0&1&0&1\end{bmatrix}$$

$$(7)\begin{bmatrix}0&1&1\\0&0&0\\0&2&1\end{bmatrix}\ (8)\begin{bmatrix}1&0&1\\0&1&1\\0&0&0\\0&0&0\end{bmatrix}\ (9)\begin{bmatrix}0&1&1&0&2\\0&0&0&1&1\\0&0&0&0&0\end{bmatrix}\ (10)\begin{bmatrix}0&1&1&-2&0\\0&0&0&1&1\\0&0&0&0&0\end{bmatrix}$$

Question 1. For the above 10 matrices, list their leading columns, and say whether they are in RREF (20 marks).

	Leading columns	In RREF?
1	none	yes
2	1,2	no
3	2	yes
4	1,3	yes
5	1,3	no

	Leading columns	In RREF?
6	1,3,4	no
7	2	no
8	1,2	yes
8	2,4	yes
10	2,4	no

Question 2. Form the augmented matrix for the following system of equations, bring to RREF using annotated Gauss-Jordan elimination **(no other method)**, and hence solve (10 marks).

$$x - y + 4z = 9, \quad 3x - y + 10z = 25, \quad x + 2z = 6$$

```
1 -1  4  9  =R1          1  0  3  8   -3*R3
3 -1 10 25  -3*R1        0  1 -1 -1   +1*R3
1  0  2  6  -1*R1        0  0 -1 -2  *(-1)  =R3

1 -1  4  9  +1*R2        1  0  0  2
0  2 -2 -2 *(1/2) =R2    0  1  0  1
0  1 -2 -3  -1*R2        0  0  1  2  in rref
```

Solution $x = 2, y = 1, z = 2$.

RREF using annotated Gauss-Jordan elimination (no other method), and hence solve (10 marks).

$-w-2x+2y+2z=3, \quad 3w+6x-6y-5z=-8, \quad -2w-4x+6y+4z=8, \quad -2w-4x+5y+7z=11$

```
-1 -2  2  2  3  *(-1) =R1    1  2  0 -2 -1   +2*R3
 3  6 -6 -5 -8   -3*R1       0  0  1  0  1
-2 -4  6  4  8   +2*R1       0  0  0  1  1   =R3
-2 -4  5  7 11   +2*R1       0  0  0  3  4   -3*R3

 1  2 -2 -2 -3                1  2  0  0  1   -1*R4
 0  0  0  1  1  swap          0  0  1  0  1   -1*R4
 0  0  2  0  2  swap          0  0  0  1  1   -1*R4
 0  0  1  3  5                0  0  0  0  1   =R4

 1  2 -2 -2 -3  +2*R2         1  2  0  0  1
 0  0  2  0  2  *(1/2) =R2    0  0  1  0  1
 0  0  0  1  1                0  0  0  1  1
 0  0  1  3  5  -1*R2         0  0  0  0  1  in rref
```

No solution.

Question 4. Form the augmented matrix for the following system of equations, bring to RREF using annotated Gauss-Jordan elimination **(no other method)**, and hence solve (10 marks).

$-w-2x+2y+2z=3, \quad 3w+6x-6y-5z=-8, \quad -2w-4x+6y+4z=8, \quad -2w-4x+5y+7z=10$

```
-1 -2  2  2  3 *(-1) =R1     1  2  0 -2 -1   +2*R3
 3  6 -6 -5 -8 -3*R1         0  0  1  0  1
-2 -4  6  4  8 +2*R1         0  0  0  1  1   =R3
-2 -4  5  7 10 +2*R1         0  0  0  3  3   -3*R3

 1  2 -2 -2 -3                1  2  0  0  1
 0  0  0  1  1  swap          0  0  1  0  1
 0  0  2  0  2  swap          0  0  0  1  1
 0  0  1  3  4                0  0  0  0  0  in rref

 1  2 -2 -2 -3 +2*R2
 0  0  2  0  2 *(1/2) =R2
 0  0  0  1  1
 0  0  1  3  4 -1*R2
```

Solutions $w=1-2r, x=r, y=1-r, z=1$, where r is arbitrary.