Maths 1263 Quiz 5 Friday 5/12/14

You may turn in your answers at the lecture on Monday 8/12. **Your answers should show all work.**

For the initial value problem

$$\frac{dy}{dx} = y^2, \quad 0 \le x \le 0.5$$
$$y(0) = 1$$

(as in quiz 4; the solution is $y = 1/(1-x)$). With stepsize $h = 0.1$,

(1) Apply the modified Euler method

$$g_1 = f(x_i, y_i)$$
$$g_2 = f(x_i + h/2, y_i + g_1 h/2)$$
$$y_{i+1} = y_i + g_2 h$$

(2) Apply the explicit trapezoidal method

$$g_1 = f(x_i, y_i)$$
$$g_2 = f(x_i + h, y_i + h g_1)$$
$$y_{i+1} = y_i + h(g_1 + g_2)/2$$

(3) Apply our best method,

$$s_1 = f(x_i, y_i)$$
$$s_2 = f(x_i + h/2, y_i + h s_1/2)$$
$$s_3 = f(x_i + h/2, y_i + h s_2/2)$$
$$s_4 = f(x_i + h, y_i + h s_3)$$
$$y_{i+1} = y_i + h(s_1 + 2s_2 + 2s_3 + s_4)/6$$

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Euler:
x[ 0] 0.000000, y[ 0] 1.000000
x[ 1] 0.100000, y[ 1] 1.100000
x[ 2] 0.200000, y[ 2] 1.221000
x[ 3] 0.300000, y[ 3] 1.370084
x[ 4] 0.400000, y[ 4] 1.557797
x[ 5] 0.500000, y[ 5] 1.800470

Modified Euler:
x[ 0] 0.000000, y[ 0] 1.000000
x[ 1] 0.100000, y[ 1] 1.110250
x[ 2] 0.200000, y[ 2] 1.247581
x[ 3] 0.300000, y[ 3] 1.423250
x[ 4] 0.400000, y[ 4] 1.655670
x[ 5] 0.500000, y[ 5] 1.977059

Explicit trapezoidal:
x[ 0] 0.000000, y[ 0] 1.000000
x[ 1] 0.100000, y[ 1] 1.110500
x[ 2] 0.200000, y[ 2] 1.248276
x[ 3] 0.300000, y[ 3] 1.424760
x[ 4] 0.400000, y[ 4] 1.658736
x[ 5] 0.500000, y[ 5] 1.983301

Best Runge-Kutta:
x[ 0] 0.000000, y[ 0] 1.000000
x[ 1] 0.100000, y[ 1] 1.111110
x[ 2] 0.200000, y[ 2] 1.249998
x[ 3] 0.300000, y[ 3] 1.428566
x[ 4] 0.400000, y[ 4] 1.666653
x[ 5] 0.500000, y[ 5] 1.999963
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