Maths 1263 Quiz 4 Friday 21/11/14

You may turn in your answers at the lecture on Monday 24/11. **Your answers should show all work.**

(1) Is $\sin(x) \quad O(1-\cos(x))$? (around $x=0$).
Is $1-\cos(x) \quad O(\sin(x))$? (Use the Taylor's Series for $\sin$ and $\cos$). I think they're in the official tables.

Answer. $\sin(x) = x - x^3/3! + x^5/5! \dots$ and $1-\cos(x) = x^2/2! - x^4/4! + x^6/6! \dots$. The dominant power of x in $\sin$ is x and in $1-\cos$ is x^2. So $\sin(x)$ is *not* $O(1-\cos(x))$, but $1-\cos(x)$ *is* $O(\sin(x))$.

(2) Calculate an approximation to

$$\log_e(1.5) = \int_{x=1}^{1.5} \frac{1}{x} dx$$

using the Trapezoidal method with 4 intervals.

Answer. The Trapezoidal Method gives 0.406187; 0.405465 is correct to 6 places, so this is correct to 2 places.

(3) Calculate an approximation to

$$\log_e(1.5) = \int_{x=1}^{1.5} \frac{1}{x} dx$$

using Simpson's Rule with 4 intervals (same data as (2)).

Answer. Simpson's Rule gives 0.405471, correct to 4 places.

(4) Give the first 4 terms – up to the x^3 term – in Taylor's series for $\log_e(1+x)$ around $x=0$. The derivative of $\log_e(1+x)$ is $1/(1+x)$.

$$f(0) = 0$$
$$f'(0) = 1$$
$$f''(0) = -1/(1+x)^2$$
$$f'''(0) = 2/(1+x)^3$$
$$x - x^2/2 + x^3/3 \dots$$

(5) Apply Euler's method to get an approximate solution to

$$\frac{dy}{dx} = y^2, \quad y(0) = 1 : 1 \le x \le 1.5$$

Correction

$$y(0) = 1; \quad 0 \le x \le 0.5$$

using steplength 0.1. (The true solution is $y = 1/(1-x)$).

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y(0.000000) = 1.000000; acc is 1.000000
y(0.100000) = 1.100000; acc is 1.111111
y(0.200000) = 1.221000; acc is 1.250000
y(0.300000) = 1.370084; acc is 1.428571
y(0.400000) = 1.557797; acc is 1.666667
y(0.500000) = 1.800470; acc is 2
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