Maths 1263 Quiz 3 answers Friday 31/10/14

You may turn in your answers at the lecture on Monday 10/11. **Your answers should show all work.**

(1: 10 marks) Use Gaussian elimination with partial pivoting to solve $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 & -7 \\ -3 & -11 & 27 \\ 2 & 6 & -13 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \text{and } B = \begin{bmatrix} 4 \\ -12 \\ 9 \end{bmatrix}$$

```
  1     3    -7     4 swap        -3   -11    27   -12
 -3   -11    27   -12 swap         1     3    -7     4   +1/3 R1
  2     6   -13     9              2     6   -13     9   +2/3 R1

 -3    -11    27   -12            -3    -11    27   -12
  0   -2/3     2     0 swap        0   -4/3     5     1
  0   -4/3     5     1 swap        0   -2/3     2     0   -1/2 R2

 -3    -11    27    -12
  0   -4/3     5      1
  0      0 -1/2   -1/2
```

Back substitution

$$z = 1$$
$$-4/3y + 5 = 1, \quad y = 3$$
$$-3x - xx + 27 = -12, \quad x = 2.$$

(2: 12 marks) (i) Apply the same operations (EROs from question 1) in the same order on a 3×3 identity matrix to produce a matrix C, (ii) apply just the *swap* operations in the same order on a 3×3 identity matrix to produce a permutation matrix P, (iii) write down P^{-1}, and (iv) let $M = C^{-1}$: calculate M.

```
(i) 1   0   0   swap     0     1     0
    0   1   0   swap     1     0     0   + 1/3 R1
    0   0   1            0     0     1   + 2/3 R1

    0   1   0            0     1     0
    1  1/3  0   swap     0    2/3    1
    0  2/3  1   swap     1    1/3    0   -1/2 R2

    0     1      0
    0   2/3      1   ..........C
```

```
    1     0   -1/2

(ii) 1   0   0   swap  0   1   0          0   1   0
     0   1   0   swap  1   0   0   swap   0   0   1    .......   P
     0   0   1         0   0   1   swap   1   0   0

(iii)   0   0   1
        1   0   0   .....P^{-1} (transpose)
        0   1   0
```

(iv) M can be calculated by solving for its entries in suitable order. The result is

$$C^{-1} = M = \begin{bmatrix} -1/3 & 1/2 & 1 \\ 1 & 0 & 0 \\ -2/3 & 1 & 0 \end{bmatrix}$$

(3: 10 marks) Calculate the LU factorisation of A *without* pivoting.

Answer. First row of U etcetera: we get

$$\begin{bmatrix} 1 & 3 & -7 \\ -3 & -11 & 27 \\ 2 & 6 & -13 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -7 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}.$$

(4: 12 marks) Calculate the LU factorisation of A *with* pivoting.

Answer. Swap the first two rows, then get the first row of U and the first column of L.

$$\begin{bmatrix} -3 & -11 & 27 \\ 1 & 3 & -7 \\ 2 & 6 & -13 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 1 & 0 \\ -2/3 & ? & 1 \end{bmatrix} \begin{bmatrix} -3 & -11 & 27 \\ 0 & ? & ? \\ 0 & 0 & ? \end{bmatrix}.$$

We now decide whether to swap rows 2 and 3. Let x represent the $(2,2)$ entry of U. Without swapping,

$$(-1/3)(-11) + x = 3: \quad x = -2/3$$

and with swapping…remember that the first column of L would be revised by swapping the second and third entries.

$$(-2/3)(-11) + x = 6: \quad x = -4/3.$$

This is larger in absolute value, so we choose to swap.

$$\begin{bmatrix} -3 & -11 & 27 \\ 2 & 6 & -13 \\ 1 & 3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ -1/3 & ? & 1 \end{bmatrix} \begin{bmatrix} -3 & -11 & 27 \\ 0 & -4/3 & ? \\ 0 & 0 & ? \end{bmatrix}.$$

There is no more swapping. We can compute the $(2,3)$ entry of U: 5; then the $(3,2)$ entry of L: $1/2$; and the $(3,3)$ entry of U: $-1/2$.

$$\begin{bmatrix} -3 & -11 & 27 \\ 2 & 6 & -13 \\ 1 & 3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ -1/3 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} -3 & -11 & 27 \\ 0 & -4/3 & 5 \\ 0 & 0 & -1/2 \end{bmatrix}.$$

(5: 6 marks) (i) Express A in terms of U and M from question (2). (ii) Express A in terms of L, U, (question 4) and P (question 2). (iii) Express L in terms of P and M (question 2).

Answers. (i) $MU = A$ (ii) $LU = PA$ (iii) $L = PM$.