Maths 1263 Quiz 1 answers Friday 3/10/14

This is an open-notes test, and you are allowed to cooperate, compare answers, etcetera. Each quiz is marked out of 50. Quizzes will contribute 20% to the mark for the module. **Your answers should show all work.**

Let $p(x)$ be the following polynomial

$$p(x) = 10x^3 - 5x^2 - 3x + 1$$

(1, 5 marks) Calculate its derivative, $p'(x)$.

$$30x^2 - 10x - 3$$

(2, 20 marks) Calculate the Sturm sequence for p.

Answer. poly and derivative

$$10x^3 - 5x^2 - 3x + 1$$
$$30x^2 - 10x - 3$$

after scaling

$$x^3 - 0.5x^2 - 0.3x + 0.1$$
$$x^2 - 0.333333x - 0.1$$

Sturm sequence

$$x^3 - 0.5x^2 - 0.3x + 0.1$$
$$x^2 - 0.333333x - 0.1$$
$$x - 0.326087$$
$$1$$

(4, 10 marks) Find pairs $i, i+1$ of consecutive integers together bracketing all the real roots of p.

```
3 sign changes at -infty: -+-+
3 sign changes at -5.000000: -+-+
3 sign changes at -4.000000: -+-+
3 sign changes at -3.000000: -+-+
3 sign changes at -2.000000: -+-+
3 sign changes at -1.000000: -+-+
2 sign changes at 0.000000: +--+
0 sign changes at 1.000000: ++++
0 sign changes at 2.000000: ++++
0 sign changes at 3.000000: ++++
0 sign changes at 4.000000: ++++
0 sign changes at 5.000000: ++++
0 sign changes at +infty: ++++
```

One sign change lost between -1 and 0, therefore one real root there. Two sign changes between 0 and 1, therefore two real roots there.

(5, 10 marks) This polynomial has an odd number of real roots. Use the Newton-Raphson method to calculate the *middle* real root correct to 6 decimal places.

Answer. Try starting Newton-Raphson at 0.

```
a = 0
0 - p(0)/p'(0) = 1/3
        This is positive --- heading in the right direction!

a=1/3
a - p(a)/p'(a) =
1/3 - (-.185185185)/(-3) = 0.271604938

a = 0.271604938
a - p(a)/p'(a) =
0.271604938 - (0.016699878)/(-3.502972108) = 0.276372283

a = 0.276372283
a - p(a)/p'(a) =
0.276372283 - (-0.000072635)/(-3.472273664) = 0.276393202

a = 0.276393202
p(a) = 0 to 9 decimal places on calculator, so it's
at calculator accuracy.
        Is it the middle root?  Visualising the graph,
        the derivative should be positive at the leftmost
        root, negative at the middle root, and positive at
        the third root. The derivative is around -3.5,
        so we do have the middle root.
```