MA1213 Quiz 07 ANSWERS 21/11/17

Rules and procedures.

1. Open notes quiz. Collaboration is o.k. **2.** Answer sheets will be collected at the end of each tutorial. **3.** Answer sheets will **not** be accepted at any other time or in any other way. **4.** Show all your work. **5.** All quizzes will be marked out of 60 and will contribute 20% to your overall mark.

Attempt 3 questions. Only the first 3 attempted will be marked.

(1). Using the notation $R, F_1, F_{1.5}$ etcetera, list all subgroups of D_8.

Answer. Subgroups have order $1, 2, 4, 8$. Notice that $R^2 = -I$ and it commutes with everything.

$$\{I\},\ \langle R\rangle,\ \langle R^2\rangle,$$
$$\langle F_1\rangle,\ \langle F_2\rangle,\ \langle F_3\rangle,\ \langle F_4\rangle,$$
$$\{I, F_1, F_2, R^2\},\ \{I, F_{1.5}, F_{2.5}, R^2\},\ D_8$$

(2). Which subgroups of D_8 are normal subgroups? (You may assume without proof that every subgroup of index 2 is a normal subgroup.)

Answer. $\{I\}, D_8$, automatically.

Since $R^2 = -I$ commutes with everything, $\{1, R^2\} \lhd D_8$.

$\langle R\rangle$, $\{I, F_1, F_2, R^2\}$,, and $\{I, F_{1.5}, F_{2.5}, R^2\}$, are normal subgroups since they have index 2.

If we conjugate F_1 by R, $RF_1R^3 = F_2$, so $\{I, F_1\}$ is not a normal subgroup. Similarly $\langle F_{1.5}\rangle$, $\langle F_2\rangle$, and $\langle F_{2.5}\rangle$, are not normal subgroups.

(3). Let $H = \langle 3\rangle \leq \mathbb{Z}_6$ (under addition). In abelian groups, every subgroup is a normal subgroup. List the left cosets of H and give the Cayley table for the quotient group $\mathbb{Z}_6/H$.

Answer.

$$\{0, 3\},\ \{1, 4\},\ \{2, 5\}$$

$+$	$\{0,3\}$	$\{1,4\}$	$\{2,5\}$
$\{0,3\}$	$\{0,3\}$	$\{1,4\}$	$\{2,5\}$
$\{1,4\}$	$\{1,4\}$	$\{2,5\}$	$\{0,3\}$
$\{2,5\}$	$\{2,5\}$	$\{0,3\}$	$\{1,4\}$

(4). The quaternion group Q_8 has eight elements $\pm 1, \pm i, \pm j, \pm k$. Implicitly $(-1)i = -i$ and so on; -1 commutes with everything. The Broombridge formula says

$$i^2 = j^2 = k^2 = ijk = -1.$$

Calculate ij, ji, and jk.

Answer.

$$(-1)k = (ijk)k = ij(-1); \qquad ij = k$$
$$i(-1) = i(ijk) = (-1)jk; \qquad jk = i$$
$$ji = j(jk) = -k$$

(5). Construct the Cayley Table for Q_8.

$\times$	1	i	j	k	-1	$-i$	$-j$	$-k$
1	1	i	j	k	-1	$-i$	$-j$	$-k$
i	i	-1	k	$-j$	$-i$	1	$-k$	j
j	j	$-k$	-1	i	$-j$	k	1	$-i$
k	k	j	$-i$	-1	$-k$	$-j$	i	1
-1	-1	$-i$	$-j$	$-k$	1	i	j	k
$-i$	$-i$	1	$-k$	j	i	-1	k	$-j$
$-j$	$-j$	k	1	$-i$	j	$-k$	-1	i
$-k$	$-k$	$-j$	i	1	k	j	$-i$	-1