# MA1213 Quiz 02 ANSWERS 10/10/17

**Rules and procedures.**

**1.** Open notes quiz. Collaboration is o.k. **2.** Answer sheets will be collected at the end of each tutorial. **3.** Answer sheets will **not** be accepted at any other time or in any other way. **4.** Show all your work. **5.** All quizzes will be marked out of 60 and will contribute 20% to your overall mark.

*Attempt 3 questions. Only the first 3 attempted will be marked.*

**(1).** Let $f : A \to B$ be a map and $R$ an equivalence relation on $B$. Let $S$ be the following relation on $A$:

$$xSy \iff f(x)\, R\, f(y)$$

Prove that the relation $S$ is an equivalence relation on $A$.

**Proof.** (i) Reflexive. For any $x \in A$, $f(x)Rf(x)$, so $xSx$.
(ii) Symmetric. Given $x, y \in A$ such that $xSy$, $f(x)Rf(y)$ so $f(y)Rf(x)$ and $ySx$.
(iii) Transitive. Given $x, y, z \in A$ such that $xSy$ and $ySz$, $f(x)Rf(y)$ and $f(y)Rf(z)$, so $f(x)Rf(z)$, so $xSz$. ∎

**(2).** If $R$ is an equivalence relation on $A$, and $f : A \to B$ a map, is the relation (viewed as a set of ordered pairs)

$$\{(f(x), f(y)) : x, y \in A \wedge xRy\}$$

an equivalence relation? If so, prove it; if not, give a counterexample.
**Answer.** No. It is symmetric and transitive but not reflexive. For instance, suppose $A = \{a, b\}$ and $B = \{1, 2\}$, and $f$ maps $a$ and $b$ both to 1. Then $(2, 2)$ is not in the given set of ordered pairs, so $(2, 2)$ is not in the relation and it is not reflexive.

**(3).** List the subsets of the set $A = \{a, b\}$. Give the sixteen values of the symmetric difference $X \vartriangle Y$ where $X, Y \subseteq A$. (Recall that $x \in X \vartriangle Y$ if and only if $x$ belongs to *exactly one* of $X, Y$.)

**Answer.** The subsets: $\emptyset, \{a\}, \{b\}, \{a, b\}$.

Table for the symmetric difference.

| | $\emptyset$ | $\{a\}$ | $\{b\}$ | $\{a,b\}$ |
|---|---|---|---|---|
| $\emptyset$ | $\emptyset$ | $\{a\}$ | $\{b\}$ | $\{a,b\}$ |
| $\{a\}$ | $\{a\}$ | $\emptyset$ | $\{a,b\}$ | $\{b\}$ |
| $\{b\}$ | $\{b\}$ | $\{a,b\}$ | $\emptyset$ | $\{a\}$ |
| $\{a,b\}$ | $\{a,b\}$ | $\{b\}$ | $\{a\}$ | $\emptyset$ |

**(4).** Now let $S$ be the set of subsets of *any* set $A$. Give an interpretation of $X \vartriangle (Y \vartriangle Z)$ similar to that in (3).
**Answer.** $x$ belongs to $X \vartriangle (Y \vartriangle Z)$ if and only if $x$ belongs to *exactly 1* or *exactly 3* of the three sets $X, Y, Z$.

**(5).** With $S$ as in (4), prove that $S$ is a group under the symmetric difference operator $\triangle$.

**Proof.** (i) Associativity: like in (4), $x$ belongs to $(X \triangle Y) \triangle Z$ if and only if $x$ belongs to just 1 or all 3; using (4), $S$ is associative.
(ii) Identity: $\emptyset$.
(iii) Inverse: $X \triangle X = \emptyset$ always. ∎