REVISION EXERCISE FOR COURSE MA3431

1. An alternative Lagrangian density for the electromagnetic field due to Enrico Fermi is

$$\mathcal{L} = -\frac{1}{2}\,\partial_\mu A_\nu\,\partial^\mu A^\nu - \frac{1}{c}\,J_\mu\,A^\mu$$

where J_μ is the four current density and the electromagnetic units are those of Heaviside and Lorentz.

   (a) Derive the Euler-Lagrange equations of motion for the alternative Lagrangian density of Fermi.

   (b) Under what assumptions are the equations of motion the Maxwell equations of electrodynamics?

   (c) Show explicitly, and with what assumptions, that the Lagrangian density of Fermi differs from

$$\mathcal{L} = -\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} - \frac{1}{c}\,J_\mu\,A^\mu$$

   by a 4-divergence.

   (d) Does the added 4-divergence affect the action relating to the electromagnetic field theory?

   (e) Does the added 4-divergence affect the equations of motion of the theory?

   [J D Jackson, Classical Electrodynamics, Problem 12.13 (2nd ed.), 12.14 (3rd ed.)].

Recommended Textbooks for Course MA3431 in Classical Field Theory

- Classical Electrodynamics, J. D. Jackson, John Wiley, 1998 (3rd ed) [537.12 K23]
- The Classical Theory of Fields, E. M. Lifshitz and L. D. Landau [530.14 L52]
- Classical Field Theory, Francis E. Low, John Wiley & Sons, 1997 [530.14 N71]
- Classical Mechanics, Herbert Goldstein, Addison-Wesley, 1980 [531 J0*1]