CLASSICAL FIELD THEORY - COURSE MA3431

Recommended texts and see the page `http://www.maths.tcd.ie/~nhb/3431.php`

- Classical Electrodynamics, J. David Jackson, Wiley, 1998 (3rd edition) [537.12 K23]
- The Classical Theory of Fields by E. M. Lifshitz and L. D. Landau [530.14 L52]
- Classical Field Theory, Francis E. Low, Wiley, 1997 (1st edition) [530.14 N71]
- Classical Mechanics, Herbert Goldstein et al., Prentice Hall, 2002 (3rd edition)

Topics will be selected from the following for the Course in Classical Field Theory

1. Lagrangian for a discrete system; Lagrangian density for a field in 1+1 dimensions
2. Hamilton's principle; symmetry and conservation laws, Noether's theorem
3. Variational principle; the conserved current; Galilean and Lorentz invariance
4. Special relativity, tensors; time reversal, space reflection; field transformations
5. Covariant field theory; free scalar field, vector fields, the four-vector potential A_μ
6. Antisymmetric field tensor $F_{\mu\nu}$; charged particle interaction and the Lorentz force
7. Lagrangian density for free vector field; Maxwell's equations for $F_{\mu\nu}$, E and B
8. Field equations with particles; gauge invariance, Lorenz gauge, charge conservation
9. Energy momentum tensor $T^{\mu\nu}$, gauge invariant, conserved, traceless; interactions
10. Symmetric gauge invariant tensor $\Theta^{\mu\nu}$, Maxwell stress tensor for fields E and B
11. Particle and field energy-momentum conservation; field angular momentum $M^{\mu\nu\sigma}$

The Context of Classical Field Theory

CM	1687	Classical Mechanics	Isaac Newton
CED	1865	Classical Electrodynamics	James Clerk Maxwell
SR	1905	Special Relativity	Albert Einstein
GR	1915	General Relativity	Albert Einstein
QM	1926	Quantum Mechanics	Schrödinger and Heisenberg
RQM	1928	Relativistic Quantum Mechanics	Paul Dirac
QED	1947	Quantum Electrodynamics	Feynman, Schwinger, Tomonaga

EXERCISES FOR SOPHISTER COURSE MA3431

1. (a) Show that the electric and magnetic induction fields transform according to Eq. (11.148) of J. D. Jackson's *Classical Electrodynamics, 3e*, using

$$F'^{\mu\nu} \;=\; \Lambda^{\mu}{}_{\rho}\,\Lambda^{\nu}{}_{\sigma}\,F^{\rho\sigma}$$

when the velocity βc of the frame K' is directed along the x^1 axis of frame K with

$$\Lambda^{\mu}{}_{\rho} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Show that the electric and magnetic fields in frame K of the electrostatic field of a stationary charge q in a frame K' moving with velocity $v = \beta c$ along the x_1 axis of frame K are given by Eq. (11.152) of J. D. Jackson's *CED, 3e*.

$$E_1 \;=\; -\frac{\gamma q v t}{(b^2 + \gamma^2 v^2 t^2)^{\frac{3}{2}}}$$

$$E_2 \;=\; \frac{\gamma q b}{(b^2 + \gamma^2 v^2 t^2)^{\frac{3}{2}}}$$

$$B_3 \;=\; \frac{\beta\gamma q b}{(b^2 + \gamma^2 v^2 t^2)^{\frac{3}{2}}}$$

with the other components vanishing, t being the time since the origins of the frames K and K' overlapped and b referring to the closest distance of approach of the charge, assumed fixed on the x_2' axis.

2. An alternative Lagrangian density for the electromagnetic field due to Enrico Fermi is

$$\mathcal{L} \;=\; -\frac{1}{8\pi}\,\partial_\mu A_\nu\,\partial^\mu A^\nu - \frac{1}{c}\,J_\mu\,A^\mu\,.$$

(a) Derive the Euler-Lagrange equations of motion. Under what assumptions are they the Maxwell equations of electrodynamics?

(b) Show explicitly, and with what assumptions, that the Lagrangian density of Fermi differs from

$$\mathcal{L} \;=\; -\frac{1}{16\pi}\,F_{\mu\nu}\,F^{\mu\nu} \;-\; \frac{1}{c}\,J_\mu\,A^\mu$$

by a 4-divergence. Does the added 4-divergence affect the action? Does it affect the equations of motion? [J. D. Jackson, Problem 12.13 (2e), 12.14 (3e)] .