Forward collisions and spin effects in evaluating amplitudes

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Abstract. Total cross sections and the phases of forward collision amplitudes form part of the early studies when a new energy window becomes available as is provided by the Large Hadron Collider. Enhancement of the forward elastic differential cross section above that expected from estimates of dispersion and optical theorem values may result from the presence of hadronic spin dependence in addition to effects induced by vacuum polarization contributions to the photon propagator. The elastic scattering of protons and ions at small angles is important in the evaluation of the luminosities of the corresponding incident beams and invites detailed examination. Polarization measurements taken at a number of high energies have yielded information on the extent of spin effects in hadronic scattering, particularly at the low momentum transfers related to diffraction.

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INTRODUCTION

As a new energy landscape becomes available at the Large Hadron Collider, total cross sections and the phases of forward hadronic amplitudes invite early measurement to verify their analytic properties [1] and to set the scale for other processes. Hadronic spin dependence in the elastic proton proton differential cross section requires study [2] if real and imaginary parts of amplitudes are to be evaluated accurately.

Vacuum polarization effects in the photon propagator [3] also induce a small known contribution to the real part of the forward amplitude as do spin effects indirectly. Luminosities for the scattering of incident protons and ions also rely upon a well considered approach to small angle scattering.

Proton proton collisions probe the analytic structure of scattering amplitudes and a dispersion relation violation may suggest a new mass scale [4] for very high energy interactions. A pioneering model due to Kaidalov and Ter Martyrosian [5] provides a context for the high energy hadronic behaviour at small collision angles.

Only some of the helicity dependent effects are kinematically suppressed near the forward direction when seeking the spin implications for evaluating the hadronic transition amplitudes of fermion fermion scattering. The influence of spin dependence for low angle scattering on cross section normalisation also needs elaboration.

The measurement of proton beam polarization at BNL RHIC [6] needed a detailed analysis of helicity nonflip and flip amplitudes for electromagnetic and hadronic interactions. Polarized protons scattering on carbon fibre and hydrogen jet targets at low momentum transfer provided a sufficiently large spin asymmetry to enable polarimetry studies to be undertaken at an acceptable statistical level.

ELASTIC SCATTERING AMPLITUDES

A new energy regime is being provided by the hadron collider at CERN. The magnitude and phases of forward collision amplitudes are amenable to analysis at much higher energies than heretofore. The spin averaged electromagnetic and hadronic amplitudes are of comparable magnitude at the interference invariant momentum transfer [7]

$$t_c \;=\; -8\,\pi\,\alpha/\sigma_{\rm tot}\,, \qquad \alpha \;=\; 1/137 \tag{1}$$

and the value of $-t_c$ is expected to decrease with increasing LHC energy where $\sigma_{\rm tot}$ may reach well over 100 mb. Higher order photon contributions are encoded in a spin independent Coulomb phase multiplying electromagnetic amplitudes with an exponent $\delta = -\,\alpha\,\ln|Bt/2 + 4t/\Lambda| - \alpha\,\gamma$ where B relates to the exponential t-behaviour of the hadronic differential cross section, Euler's constant is $\gamma \approx 0.5772$, and $\Lambda = 0.71\,{\rm GeV}^2$ reflects the small momentum transfer dependence of the electromagnetic form factors

$$G_E(t) \;\approx\; G_M(t)/\mu \;\approx\; (1-t/\Lambda)^{-2}\,. \tag{2}$$

with $\mu = 2.793$ as the magnetic moment of the proton. Observe that in proton proton elastic collisions, by contrast with spin scalar scattering, the high energy element $(\alpha\,/\,t\,)\,(s-2m^2)\,F_1{}^2(t)$ of the spin averaged electromagnetic amplitude [8] involves a Dirac form factor which has the following expansion in small $-t$,

$$F_1(t) \;=\; \frac{G_E \,-\, t\,G_M/4m^2}{1\,-\,t/4m^2} \;\approx\; 1-\left(\frac{\mu-1}{4m^2}-\frac{2}{\Lambda}\right)t \tag{3}$$

so that an extra term appears in the interference element of the expression [9] describing the differential cross section for elastic proton proton collisions in the interference region [10]. An expansion including singular terms in t and constant terms may be written [11]

$$\begin{aligned} \frac{16\pi}{\sigma_{\rm tot}^2}\,\frac{d\sigma}{dt}\,e^{-Bt} \;&=\; \frac{t_c^2}{t^2} \,-\, 2\,(\rho+\delta+\varepsilon\,)\,\frac{t_c}{t} \,+\, 1+\rho^2 \\ &-\;\; 2t\left[\left(\tfrac{1}{2}\,\kappa\,t_c/t-{\rm Re}\,r_5\right)^2 + \left({\rm Im}\,r_5\right)^2\right]\Big/m^2 \\ &+\;\; \left[2\,\Delta\sigma_{\rm T}^2\left(1+\rho_2^2\right) \,+\, \Delta\sigma_{\rm L}^2\left(1+\rho_-^2\right)\right]/\,4\,\sigma_{\rm tot}^2 \end{aligned} \tag{4}$$

where δ is the Coulomb phase and $\kappa = 1.793$ is the anomalous moment of the proton. The extra interference term incorporates the hadronic slope

$$\varepsilon \;=\; \left(\frac{B}{2}-\frac{4}{\Lambda}+\frac{\mu-1}{2m^2}\right)t_c \;\approx\; \frac{B-9.23}{2}\,t_c\,. \tag{5}$$

The last term of Eq. (4) has been studied at lower GeV energies [12]. It is a sum of forward transverse and longitudinal hadronic double helicity flip contributions involving respective real part to imaginary part parameters ρ_2 and ρ_- as well as the polarized total cross section differences

$$\Delta\sigma_{\rm T} \;=\; \sigma_{\uparrow\downarrow} \,-\, \sigma_{\uparrow\uparrow}\,, \qquad \Delta\sigma_{\rm L} \;=\; \sigma_{\overset{\rightarrow}{\leftarrow}} \,-\, \sigma_{\overset{\rightarrow}{\rightarrow}}\,. \tag{6}$$

SPIN DEPENDENT EFFECTS

As a fraction of the forward imaginary spin averaged amplitude, the kinematically scaled single helicity flip hadronic amplitude r_5 [11] has been bounded using $A_{\rm N}$ measurements resulting from a polarised jet study of the single spin asymmetry $A_{\rm N}$ at RHIC [13].

$$|\,r_5\,| \;<\; 0.11 \pm 0.08 \quad (6.8\,\text{GeV}); \quad 0.02 \pm 0.03\,(13.7\,\text{GeV})\,. \tag{7}$$

Other measurements taken at Fermilab [14] and at Brookhaven National Laboratory [15] have indicated that the absolute value of the single helicity flip hadronic amplitude at a number of energies is limited by

$$|\,r_5\,| \;<\; 0.15 \pm 0.26\,(19.4\,\text{GeV}); \quad 0.44 \pm 0.42\,(\,200\,\text{GeV}). \tag{8}$$

Neglecting then this apparently small single helicity flip hadronic amplitude, but keeping the corresponding singular electromagnetic flip amplitude, the differential cross section near the electromagnetic-hadronic interference region can take the form

$$\frac{16\pi}{\sigma_{\rm tot}^2}\,\frac{d\sigma}{dt}\,e^{-Bt} \;=\; \frac{t_c^2}{t^2} \,-\, 2\,(\,\rho+\delta+\varepsilon'\,)\,\frac{t_c}{t} \,+\, 1+\rho^2 \tag{9}$$

$$+\;\; \left[2\,\Delta\sigma_{\rm T}^2\left(1+\rho_2^2\right) \,+\, \Delta\sigma_{\rm L}^2\left(1+\rho_-^2\right)\right]/\,4\,\sigma_{\rm tot}^2 \tag{10}$$

where, incorporating the single helicity flip electromagnetic amplitude, an extra term is

$$\varepsilon' \;=\; \left(\frac{B}{2}-\frac{4}{\Lambda}+\frac{\mu^2-1}{4m^2}\right)t_c \;=\; \frac{B-7.40}{2}\,t_c\,. \tag{11}$$

The extra term $(\mu^2-1)t_c/4m^2 = 1.93\,t_c$ would contribute to an accurate determination of ρ. Transverse spin total cross section differences have been found at various energies

$$\Delta\sigma_{\rm T} \;=\; \begin{cases} -1.26 \pm 0.88\ \text{mb} \ (\ 6.8\ \text{GeV}) \\ \ \ 0.02 \pm 0.23\ \text{mb} \ (13.7\ \text{GeV}) \\ -0.04 \pm 0.50\ \text{mb} \ (200\ \text{GeV}) \end{cases} \tag{12}$$

and also the absolute values of transverse contributions are available from very recent measurements of the double spin asymmetry $A_{\rm NN}$ at BNL RHIC [13]

$$\frac{\Delta\sigma_{\rm T}}{\sigma_{\rm tot}}\left(1+\rho_2^2\right)^{1/2} \;<\; \begin{cases} 0.103 \ (\ 6.8\ \text{GeV}) \\ 0.017 \ (13.7\ \text{GeV})\,. \end{cases} \tag{13}$$

Enhancement of the forward elastic differential cross section above that expected from estimates of dispersion and optical theorem values may result from the presence of hadronic spin dependence. In addition to the above transverse effects, contributions from a longitudinal total cross section difference measured at Fermilab [16]

$$\Delta\sigma_{\rm L} \;=\; 0.040 \pm 0.048 \pm 0.052\ \text{mb}\ (200\ \text{GeV}) \tag{14}$$

are negligible at 200 GeV/c but little is currently known about the corresponding real part or the values at higher energies. Further enhancement of the cross section can result from a consideration of vacuum polarization contributions to the photon propagator.

VACUUM POLARIZATION EFFECTS

An electron loop in the photon propagator induces a vacuum polarization contribution [3] that increases the coupling α to $\alpha(1+\Delta\nu)$ with a t-dependence given by Källén [17]

$$\Delta\nu = \frac{\alpha}{3\pi}\left[\frac{1}{3} + \left(3 - \frac{1}{\tau}\right)\left(\frac{1}{3}\tau + \frac{1}{5}\tau^2 + \frac{1}{7}\tau^3 + \cdots\right)\right] \tag{15}$$

in the case $|\tau| < 1$ where $\tau^{-1} = 1 - 4m_e^2/t$. For $-t \gg m_e^2$, the term $\Delta\nu$ has t-dependence

$$\Delta\nu \;=\; \frac{\alpha}{3\pi}\left(\ln\left|\frac{t}{m_e^2}\right| - \frac{5}{3} - \frac{6m_e^2}{t} + \cdots\right). \tag{16}$$

For $0 < -t \ll m_\mu^2$ negligible contributions result from muon and higher mass pairs. At $\sqrt{s} = 546$ GeV, for example, the vacuum polarization contribution to the real part parameter $\rho(\mathrm{pp})$ would be $\Delta\nu = 0.005$ in the interference region of proton proton elastic scattering and have the opposite sign for elastic antiproton proton collisions.

In conclusion, the analyticity of elastic scattering amplitudes can probe a possible high mass scale. Polarization measurements now indicate the magnitude of hadronic helicity dependence. Spin and vacuum polarisation contributions may play a rôle in the detailed low momentum transfer studies important for the understanding of diffraction.

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