Form Factors from Polarised Proton Antiproton Annihilation to Massive Lepton Pairs

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Abstract

General expressions for single and double spin asymmetries of the lepton antilepton to proton antiproton annihilation process are given that include the mass of the lepton. Polarisation observables yield much information on the phases and absolute values of the complex nucleon electromagnetic form factors G_E and G_M in the time-like region. Such information is also accessible from asymmetries in the time reversed antiproton proton to lepton antilepton reactions thus clarifying the nature of nucleon structure.

1 Introduction

The form factors of nucleons as measured in space like and time like domains provide fundamental information on hadronic structure and internal dynamics. Scattering of high energy electrons by protons make it possible to determine the proton form factors in the region of space like momentum transfer ($q^2 < 0$). Information on the time like form factors is accessible through annihilation reactions such as $\bar{p}\,p \to l^+\,l^-$ where lepton, l, refers to the electron, muon or tau lepton.

There is great theoretical interest in the nucleon time like form factors due to recent experiments that have raised serious issues. Values for the proton have been obtained over many years via electron proton scattering, often using the Rosenbluth separation technique [1]. The magnetic proton form factor has been measured at q^2 values up to 31 GeV^2 in the space like region [2] and from $\bar{p}\,p$ or e^-e^+ annihilation up to $q^2 = 5.6\,\text{GeV}^2$ [3].

Recent measurements of the electron to proton polarisation transfer [4, 5] in $e^-\,p \to e^-\,p$ scattering at Jefferson Laboratory show that the ratio of Sachs

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form factors $G_E(q^2)/G_M(q^2)$ is monotonically decreasing with increasing q^2 in contradiction with the G_E/G_M scaling assumed in the Rosenbluth separation method.

Some conjecture that data on G_E in the space like region extracted by the Rosenbluth technique may be unreliable and should be ignored in a global analysis [6]. The Rosenbluth method may also be considered incomplete in the space like region because of its sensitivity to uncertain radiative corrections, including two-photon exchange effects [7]. A recent analysis indicates that radiative corrections are too small to explain the discrepancy [8].

In the time like region, also, measurements at Fermilab [3] have unexpectedly shown that $|G_M|$ is twice as large as in the space like region. Although the space like form factors of a stable hadron are real, the time like form factors have a phase reflecting the final state interaction of the outgoing hadrons in a reaction such as $e^+ e^- \to p\,\bar{p}$. Such interactions may be responsible for the enhancement of $|G_M|$ in the time like region [9].

Polarisation measurements and a precise separation of form factors are planned at the future antiproton facility at GSI [10]. The Polarised Antiproton eXperiment (PAX) collaboration seeks to generate polarised antiprotons by spin filtering with an internal polarised gas target. The origin of the unexpected q^2 dependence of the ratio G_E/G_M of the proton can be clarified by a measurement of their relative phase in the time like region discriminating strongly between models of the form factor. This phase can be measured via a single spin asymmetry in the annihilation reaction $p\,\bar{p} \to e^- e^+$ on a transversely polarised target [11].

The proposed measurement of this phase at PAX will also contribute to the understanding of the onset of the pQCD asymptotics in the time like region and will serve as a stringent test of dispersion theory approaches to the relationship between the space like and time like form factors.

The double spin asymmetry will limit the relative phase ambiguity and allow independent G_E—G_M separation, serving as a check on the Rosenbluth separation in the time like region. Despite the fundamental implications of the phase for the understanding of the connection between the space like and time like factors, such measurements have yet to be accurately obtained.

The Dirac and Pauli form factors, F_1 and F_2, are analytic functions of q^2 and take real values in the space like region $q^2 < 0$ due to the Hermiticity of the electromagnetic Hamiltonian. In the time like region the form factors are complex on the real q^2 axis above threshold due to unitarity and time reversal invariance. Form factors in the time and space like regimes are related by dispersion relations [12, 13].

Measurements are particularly important at high momentum transfers as they serve to test the predictions of perturbative QCD.

2 Unpolarised cross section

The differential cross section for proton-antiproton annihilation leading to a lepton-antilepton pair involves $s = q^2 = q_\mu q^\mu$ the square of the invariant total 4-momentum. In the metric $q^2 = q_0^2 - \mathbf{q}^2$, the invariant q^2 is positive in the time like region and is given in terms of the final proton and antiproton momenta by

$$q^\nu \;=\; P^\nu + P'^{\,\nu} \,. \tag{1}$$

We assume that the lepton has no structure and study the time reversed process $l^- l^+ \rightarrow p\,\bar{p}$, with M being the mass of the proton and where the momenta of the incoming lepton and antilepton are K^μ and $K'^{\,\mu}$ respectively. The momenta P^ν and $P'^{\,\nu}$ refer to the outgoing proton and antiproton.

The form factors, F_1 and F_2, are functions of q^2 and normalised such that $F_1(0) = 1$ and $F_2(0) = \mu_p - 1$ where μ_p is the magnetic moment of the proton. Equal at threshold ($s = 4\,M^2$), the Sachs electric and magnetic form factors are given by [14]

$$\begin{aligned} G_E(s) \;&=\; F_1(s) + \frac{s}{4\,M^2}\,F_2(s) \\ G_M(s) \;&=\; F_1(s) + F_2(s)\,. \end{aligned} \tag{2}$$

We note here that while for t channel scattering ($q^2 < 0$) the variables $G_E(q^2)$ and $G_M(q^2)$ are real on the negative real q^2 axis, the form factors can take complex values for $q^2 > 4\,m_\pi^2$, where m_π is the mass of the pion [15].

The spin averaged differential cross section for $l^+ + l^- \rightarrow p + \overline{p}$ scattering in terms of Mandelstam varibles s and $t = (P - K)^2$ is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{s^3\,(s - 4\,M^2)} \left\{ \frac{s^2}{2}\left(s - 4\,M^2\right)|G_M|^2 - 4\,s\,m^2 M^2 \left(|G_M|^2 - |G_E|^2\right) \right.$$
$$\left. + \left[\left(t - m^2 - M^2\right)^2 + s\,t \right] \left(s\,|G_M|^2 - 4\,M^2 |G_E|^2 \right) \right\} \tag{3}$$

where m is the mass of the lepton. The flux factor $\beta = \sqrt{s - 4M^2}/\sqrt{s - 4m^2}$. This formula agrees with that first obtained in Ref. [16] in the case of zero lepton mass. The absolute values of the form factors can be determined by a Rosenbluth separation technique. Measurements are made at at a fixed value of the total energy q^2 and at a number of centre of mass scattering angles θ given in Eq. 5 for nonzero lepton mass. The θ-dependence is of the $\cos^2\theta$ form expected [17].

3 Asymmetries with lepton mass

The invariant Mandelstam variables are s, t, u, with $u = 2m^2 + 2M^2 - s - t$. In terms of such variables the scattering angle for the process $\bar{p}\,p \to l^+\,l^-$ in the centre of mass system is [18]

$$\cos\theta = \frac{t - u}{\sqrt{s - 4\,m^2}\,\sqrt{s - 4\,M^2}} \tag{4}$$

$$\sin\theta = \frac{2\left[4\,m^2\,M^2 - \left(t - m^2 - M^2\right)^2 - s\,t\right]^{1/2}}{\sqrt{s - 4\,m^2}\,\sqrt{s - 4\,M^2}}\,.$$

We shall retain the lepton mass for the reaction $l^+\,l^- \to p\,\bar{p}$ and derive the general expressions for the single and double spin observables. It will be convenient to define a scaled unpolarised cross section in the centre of mass system

$$\frac{\sqrt{s - 4m^2}\sqrt{s - 4M^2}}{4\,\pi}\,\frac{d\sigma}{dt} = \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4}\,\frac{\beta}{s}\,D \tag{5}$$

where D may be given in terms of the centre of mass scattering angle, θ

$$D = \left(1 - \frac{4m^2}{s}\right)\left[\frac{4M^2}{s}|G_E|^2 - |G_M|^2\right]\sin^2\theta + 2\left[\frac{4m^2M^2}{s^2}|G_E|^2 + |G_M|^2\right].$$

This general expression for the spin averaged cross section reduces to a known result [16] when the lepton mass $m \to 0$.

It has been noted that it is possible to define an angular asymmetry, R, [19] which can be measured from the differential cross section at $\theta = \pi/2$. This measurement does not require polarised particles and R is defined in terms of the form factors as

$$\mathsf{R} \;=\; \frac{s\,|G_M|^2 - 4\,M^2\,|G_E|^2}{s\,|G_M|^2 + 4\,M^2\,|G_E|^2}\,. \tag{6}$$

The three polarisation orientations used in the following are called longitudinal, in the scattering plane and normal but often denoted z, x and y, respectively. In the scattering plane, (x), is perpendicular to the direction of the outgoing baryon. Longitudinal, (z), means parallel to the direction of the outgoing baryon. Normal, (y), refers to normal to the scattering plane in the direction of $\mathbf{p} \times \mathbf{k}$ where $\mathbf{k}$ is the lepton momentum and $\mathbf{p}$ is the proton momentum, with x, y and z forming a right handed coordinate system.

4 Single spin asymmetry

In order to determine the relative phase of the form factors it is necessary to perform experiments with polarised protons or antiprotons. When either the proton or the antiproton in the reaction $l^+\,l^- \to p\,\bar{p}$ is polarised, it is possible to measure the asymmetry parameter A_y defined as

$$A_y \;=\; \frac{(d\sigma/d\Omega)_\uparrow \;-\; (d\sigma/d\Omega)_\downarrow}{(d\sigma/d\Omega)_\uparrow \;+\; (d\sigma/d\Omega)_\downarrow} \tag{7}$$

where the subscripts $\uparrow$ and $\downarrow$ refer to the polarisations in the y direction. The general single spin asymmetry for either a polarised proton or a polarised antiproton of mass M annihilating to a lepton antilepton pair, each of mass m, is

$$A_y \;=\; \left(1 - \frac{4\,m^2}{s}\right)\,\frac{2\,M\,\sin 2\theta}{\sqrt{s}\;D}\,\mathsf{Im}\,G_E^\star\,G_M \tag{8}$$

where θ is the centre of mass scattering angle, from Eq. 5 a function of the lepton mass. The observable vanishes at $\theta = \pi/2$ due to the factor $\sin 2\theta$ in A_y [20]. The predicted single spin asymmetry is substantial and has a distinct

q^2 dependence which strongly discriminates between the analytic forms which fit the proton G_E/G_M data in the space like region. As emphasised already [16], knowledge of the phase difference between G_E and G_M may strongly constrain models for the form factors.

Neglecting the lepton mass in the expression given in Eq. 8 for the asymmetry A_y provides a result that is in agreement with Ref. [16, 19, 21].

5 Double spin asymmetries

If we now consider both the outgoing proton and antiproton as polarised we arrive at the following full expressions for the double spin observables A_{xx}, A_{yy} and A_{zz} including lepton mass

$$D\,(1 - A_{xx}) = 2\,|G_M|^2 - 2\left(1 - \frac{4\,m^2}{s}\right)\sin^2\theta\,|G_M|^2 \tag{9}$$

$$D\,(1 - A_{yy}) = 2\,|G_M|^2 \tag{10}$$

$$\begin{aligned} D\,(1 - A_{zz}) &= \frac{8}{(1 - s/4\,M^2)\,s^2}\left\{\left[\left(t - m^2 - M^2\right)^2 + s\,t\right]|G_E|^2 \right.\\ &\qquad \left. + s\,m^2\left(|G_M|^2 - |G_E|^2\right)\right\}. \end{aligned} \tag{11}$$

The double spin asymmetry A_{xy} is zero in the one photon exchange approximation although this is not true in general. For example, when considering the two photon contribution mechanism this spin observable is non-zero [7].

The asymmetries A_{xz} and A_{zy} involving unlike spin directions can be expressed as

$$A_{xz} = \frac{2\,M}{\sqrt{s}\,D}\left(1 - \frac{4\,m^2}{s}\right)\sin 2\theta\;\mathrm{Re}\,G_E^\star\,G_M \tag{12}$$

$$A_{zy} = \frac{2\,M}{\sqrt{s}\,D}\left(1 - \frac{4\,m^2}{s}\right)\sin 2\theta\;\mathrm{Im}\,G_E^\star\,G_M\,. \tag{13}$$

All of the above formulae (9)–(13) reduce to previously published expressions [7] for double spin observables when the lepton mass tends to zero.

The above equations indicate that polarisation observables can be used to evaluate the relative phases of the time like form factors. Most of the double spin observables depend on the moduli squared of the form factors apart from the asymmetries A_{xz} and A_{zy} which relate to the real and imaginary parts, respectively.

6 Conclusions

The understanding of the electromagnetic structure of the nucleon, as revealed in proton antiproton reactions, is of upmost importance in any theory or model of strong interactions. Abundant data over a large range of momentum transfer already exist and we have provided an overview of the rôle of the lepton mass in a study of the nucleon electromagnetic form factors in the time like region.

General expressions, including the lepton mass, for the spin averaged cross section as well as single and double spin asymmetries have been presented here. For the annihilation reaction $\bar{p}\,p \to l^- l^+$ we simply invert the flux factor β in Eq. 3 to obtain the unpolarised differential cross section.

Previously published results for polarisation observables in the positron electron to nucleon antinucleon reaction have been given in the case of zero lepton mass. Proposed polarised antiproton experiments by the PAX collaboration in the time like region will examine the moduli and relative phases of the form factors of the proton over a range of energies. Knowledge of these phases will make it possible to separate the magnetic and electric form factors in the time like region and thus permit significant tests of QCD and the asymptotic domain. In the case of muon and tauon final state pairs it will be necessary to retain the lepton mass in the formulae for both the spin averaged differential cross section and the polarisation observables in antiproton proton annihilation processes.

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