Asymmetry and Peripheral Spin Dependence

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Abstract. Impressive bounds on the magnitude of spin dependent hadronic amplitudes have been obtained from values of the analyzing power and asymmetries provided by studies of low angle proton elastic scattering at the Relativistic Heavy Ion Collider. A bound on the single flip amplitude does rely upon knowing other amplitudes sufficiently accurately, particularly their phases, including the Coulomb phase. An examination of the maximum of the asymmetry $A_N$ suggests evaluating the important helicity dependent hadronic amplitudes through further measurements of spin observables in the low momentum transfer region at a number of energies and over a range of nuclei of different spins. The possible use of the dynamics of storage rings in this endeavour is outlined.

Keywords: Nucleon, polarization, spin observables, helicity amplitudes


Introduction

A study of polarization observables in the low momentum transfer region provides information on high energy spin dependence in hadronic elastic and diffractive processes. At the partonic spin structure frontier, elastic hadronic and electromagnetic interactions may be probed by the study of peripheral longitudinal and transverse spin asymmetries and polarization transfers.

Measurements of the single spin analyzing power for near forward proton proton collisions at energies of $\sqrt{s} = 13.7$ GeV [1] and $\sqrt{s} = 200$ GeV [2] at RHIC have provided interesting information on the energy dependence of a hadronic helicity flip amplitude.

Polarized protons [3] facilitate the study of spin dependent amplitudes particularly imaginary parts involving the difference between the parallel and antiparallel transversely and longitudinally polarized proton proton total cross sections as indicated

$$\phi_+(s,0) \propto \sigma_{\text{tot}}(i + \rho), \quad \phi_-(s,0) \propto \Delta \sigma_L(i + \rho),$$

$$\phi_5(s,t) \propto r_5 \sqrt{-t}, \quad \phi_2(s,0) \propto \Delta \sigma_T(i + \rho_2)$$

(1)

with the definitions, $\phi_\pm = (\phi_1 \pm \phi_3)/2$, where $\phi_\pm$ refers to the spin averaged elastic proton proton amplitude [4]. Ratios of helicity amplitudes relative to the dominant imaginary amplitude $\text{Im} \phi_+$, with a proton of mass $m$ and magnetic moment $\mu = \kappa + 1$

$$R_2+iL_2 = \frac{\phi_2}{2 \text{Im} \phi_+}, \quad R_-+iL_- = \frac{\phi_-}{\text{Im} \phi_+}, \quad R_5+iL_5 = \left( \frac{m}{\sqrt{-t}} \right) \frac{\phi_5}{\text{Im} \phi_+}$$

(2)

are expected to have a less pronounced dependence on the momentum transfer $t$. The amplitude $\phi_4$ may be neglected here due its proportionality to a kinematic factor $t$. 

TABLE 1. The coefficients $a_j$ and $b_j$ for $pp$ spin observables.

<table>
<thead>
<tr>
<th>Observable</th>
<th>$a_j$</th>
<th>$b_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_0$</td>
<td>$\rho$</td>
<td>$\frac{(1 + \rho^2 + R_-^2 + I_-^2) / 2 + R_-^2 + I_-^2}{2}$</td>
</tr>
<tr>
<td>$A_{NN}\Sigma_0$</td>
<td>$R_-$</td>
<td>$R_5 (\rho + R_-) + I_5 (1 + I_-)$</td>
</tr>
<tr>
<td>$A_{LL}\Sigma_0$</td>
<td>$R_-$</td>
<td>$\rho R_- + L_+ + R_5^2 + I_5^2$</td>
</tr>
<tr>
<td>$-A_{SL}\Sigma_1$</td>
<td>$\kappa (R_5 + R_-) / 2$</td>
<td>$R_5 (R_5 + R_-) + I_5 (I_5 + I_-)$</td>
</tr>
<tr>
<td>$-A_N\Sigma_1$</td>
<td>$I_5 - \kappa (1 + I_2) / 2$</td>
<td>$I_5 (\rho + R_-) - R_5 (1 + I_2)$</td>
</tr>
</tbody>
</table>

Spin Asymmetries

The observables for spinor particle elastic scattering $A_N, A_{NN}, A_{SS}, A_{LL}, A_{SL}$ have singular terms in $t$ arising from photon exchange. A scaled $pp$ differential cross section $\Sigma_0$ with slope parameter $B$ [4]

$$\Sigma_0 \equiv \frac{t}{\sigma_{tot} e^{2Bi}} \frac{d\sigma}{dt} = \frac{4\pi}{\sigma_{tot}} \frac{\alpha^2}{t} + \alpha a_0 + \frac{\sigma_{tot}}{8\pi} b_0 t + \cdots$$

(3)

has an expansion in increasing powers of $t$ with constant coefficients $a_0$ and $b_0$. For initial protons polarized along an axis $N$ normal to the scattering plane (and similarly for $LL$ longitudinal) the expansion is

$$A_{NN}\Sigma_0 = \alpha a_{NN} + \frac{\sigma_{tot}}{8\pi} b_{NN} t + \cdots.$$  

(4)

Spin observables with initial protons spins aligned along perpendicular axes $S$ and $L$ have a similar power series expansion

$$A_{SL}\Sigma_1 = \alpha a_{SL} + \frac{\sigma_{tot}}{8\pi} b_{SL} t + \cdots,$$

where it is convenient to define another scaled differential cross section quantity $\Sigma_1$. In identical $pp$ scattering, $A_{LS} = A_{SL}$, and near the forward direction the expected equality $A_{NN} \approx A_{SS}$ could be readily checked [5]. Like $A_{SL}$, the spin asymmetry $A_N$ with one of the initial protons polarized has a $\sqrt{-t}$ factor and has expansion

$$A_N\Sigma_1 = \alpha a_N + \frac{\sigma_{tot}}{8\pi} b_N t + \cdots.$$  

(6)

The ten expressions for the coefficients $a_i$ and $b_i$ of the asymmetry expansions exhibited in Table 1 are almost sufficient to determine the values of the hadronic quantities

$$\rho, R_j, I_j, \quad j = -, 2, 5,$$

(7)

or at least provide bounds on the $pp$ amplitudes. For $A_N(pp)$, the occurrence in the coefficient $a_N$ of both $I_5 = \text{Im } r_5$ and $I_2$ obstructs the unambiguous evaluation of the
helicity flip amplitude, particularly $I_5$ as the quantity $I_2 = -\Delta \sigma_T/2 \sigma_{tot}$ is not well known, being related to a $pp$ transverse total cross section difference. This is not a difficulty in proton carbon elastic scattering since double helicity amplitudes are absent for spinless targets. The proton carbon helicity flip amplitude can be directly evaluated.

**Polarization Evolution**

Understanding the gain of polarization induced by spin filtering via beam loss in storage rings requires detailed knowledge of spin observables for their explanation. The reverse may be illuminating. Studies of the decreases in luminosity and increases in polarization over a range of machine acceptance angles can provide values for asymmetry and polarization transfer observables at unusually low scattering angles. For example [6],

$$\frac{d}{d\tau} \begin{bmatrix} N \\ J \end{bmatrix} = -n \nu \begin{bmatrix} I_c - I_a & P(A_c - A_a) \\ P(A_c - K_a) & I_c - D_a \end{bmatrix} \begin{bmatrix} N \\ J \end{bmatrix}$$

(8)

describes the time $\tau$ rate of change of the number of beam particles $N(\tau)$ and their total spin $J(\tau)$ when circulating at frequency $\nu$ through a polarised target of areal density $n$ and polarisation $P$ normal (or longitudinal) to the ring plane. The loss of particles from the beam involves as coefficient the difference between complete (c) and accepted (a) quantities, the first element of the rate matrix [7], integrated over all angles beyond $\theta_a$

$$I_c - I_a = 2 \pi \int_{\theta_a}^{\pi} \frac{d\sigma}{d\Omega} \sin \theta d\theta$$

(9)

where $\theta_a$ is the acceptance angle of the accelerator ring. A second change in the beam results from a product of the target polarisation $P$ with the azimuthal average of the transverse double spin asymmetry integrated over angles beyond acceptance

$$A_c - A_a = \pi \int_{\theta_a}^{\pi} (A_{NN} + A_{SS}) \frac{d\sigma}{d\Omega} \sin \theta d\theta .$$

(10)

A change in the total spin $J$ involves particles remaining in the ring and hence a product of $P$ with the average spin transfer observable integrated up to acceptance (a) over angles above a minimum angle $\theta_0$ related to the average distance between charges, $-\theta_0 = n^2$, an impact parameter beyond which scattering is inhibited

$$K_a = \pi \int_{\theta_0}^{\theta_a} (K_{NN} + K_{SS}) \frac{d\sigma}{d\Omega} \sin \theta d\theta$$

(11)

Another contribution to the change in $J$ comes from $P$ times the averaged transverse asymmetry observable integrated completely (c) from $\theta_0$. A contribution to a change in $J$ involving $J$ itself, part of the fourth element, results from the averaged depolarisation observable, $D_a$ (with a formula similar to that of $K_a$) in the form of a loss of polarisation quantity integrated below acceptance, namely, $L_a = (I_a - D_a)/2$. Introduced by
Takakazu Seki in 1683, determinants reveal the discriminant for the eigenvalues here as

$$L_d = \sqrt{P^2 (A_c - A_a) (A_c - K_a) + L_a^2}.$$  \hspace{1cm} (12)

In the short term, the rates of change of luminosity and polarisation are approximately

$$dN/d\tau \approx n v P (I_a - I_c), \hspace{1cm} d(J/N)/d\tau \approx n v P (K_a - A_c),$$ \hspace{1cm} (13)

while in the longer term, the polarisation $J/N$ increases in magnitude to the limit [8]

$$\lim_{\tau \to \infty} J/N = P (K_a - A_c) / (L_a + L_d).$$ \hspace{1cm} (14)

Such quantities relate to integrals over double spin asymmetries that have singular behaviour in $t$ and are enhanced [9]. Normal and longitudinal orientations of the target polarization at varying acceptance angles lead to different rates of change and provide information on the values of $\rho$, $R_2$, $I_2$, $R_\perp$, $I_\perp$, $R_5$, and $I_5$ for proton proton reactions.

**Conclusions**

Spin dependent hadronic collisions offer indicative tests of nonperturbative QCD. The maximum analyzing power at interference has probed spin effects effectively and double spin asymmetries near the forward direction are expected to reveal further insights. The kinetics of stored proton polarization is another potential source of information on hadronic spin dependence. There is the long term hope of initiating a causality test via the use of the analytic structure of forward spin dependent amplitudes particularly if progress is achieved in securing polarized antiprotons of sufficient intensity and energy. Such a programme will lead to a deeper understanding of hadronic spin dynamics.

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**REFERENCES**