LHCspin and Polarimetry

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A study of forward proton proton spin dependence is provided and its implications for the high energy behaviour of amplitudes relating to polarimetry are outlined - particularly in the CNI (Coulomb nuclear interference) momentum transfer region of collision

OUTLINE

- Polarised p, d, and ^{3}He beams provide polarised up and down quarks

  Musgrave *et al.*, PoS PSTP **2017** (2018) 020

- Polarimety for a fixed polarised target can use L–R recoil asymmetries
  - examine helicity amplitudes for p p↑ and p ^{3}He↑ elastic reactions
  - the analysing power in the CNI region of $q^2 = -t$ can reach 4.5 %

- Asymmetries in the electromagnetic hadronic interference region

  Kopeliovich and Lapidus, Yad Fiz 19 (1974) 340

- Express $A_{\rm N}$ and $A_{\rm NN}$ in terms of values of the hadronic ratios r_5 and r_2

- Analysis of exchanges at 100 and 255 GeV leads to asymmetry prediction

Elastic Amplitudes and CNI Analysing Powers

Neglecting the Coulomb phase δ_C, the real part ratio ρ, hadronic spin-flip and form factor effects, the spin nonflip f and spin-flip g amplitudes are

$$f = \frac{\alpha}{t} + i\,\frac{\sigma_{\rm tot}}{8\,\pi} \qquad \text{and} \qquad g = \frac{\mu - 1}{2}\,\frac{q}{m}\,\frac{\alpha}{t}$$

indicating that the analysing power reaches an extremum at momentum transfer $t_e = -8\sqrt{3}\,\pi\,\alpha/\sigma_{\rm tot}$ arising from the following $q^2 = -t$ dependence

$$A_{\rm N} = \frac{2\,{\rm Im}\,(\,f^*\,g\,)}{|\,f\,|^2 + |\,g\,|^2} \;=\; \frac{\mu - 1}{m}\,(-3\,t_e)^{1/2}\,\frac{(t/t_e)^{3/2}}{3\,(t/t_e)^2 + 1}$$

where $|\,g\,|^2$ has been ignored relative to the larger $|\,f\,|^2$ in the CNI region. The extreme value of the pp$\rightarrow$pp analysing power in the interference region

$$A_{\rm N}^{\rm e} \;=\; (\mu - 1)\,(-3\,t_e)^{1/2}/4\,m \;\approx\; 4.5\,\%$$

Kinematics for p p↑ Elastic Collisions

- Polarimetry at HJET BNL used $\mathrm{p}\!\uparrow \mathrm{p}\!\uparrow \to \mathrm{p\,p}$ for 100 GeV and 255 GeV
- Best results occurred using recoil kinetic energies: $2.0 < T_{\mathrm{R}} < 5.5$ MeV
- Systematic errors below 2 MeV & inelastic events above 5.5 MeV intruded
- Peak analysing power appeared at $T = 1.4$ MeV, $t = -0.0015$ (GeV$/c)^2$
- For 255 GeV, $\sigma_{\mathrm{tot}} = 39.2$ mb, and at 7 TeV, $\sigma_{\mathrm{tot}} \approx 47$ mb, a factor 1.2
- Recoil energies for 7 TeV may be best for values: $1.7 < T_{\mathrm{R}} < 4.6$ MeV
- Recoil angles measured from 90 degrees would be: $30 < \theta < 50$ mrad

Amplitudes and Asymmetries for p ^{3}He Elastic Collisions

$$
\begin{aligned}
\phi_1 &= \langle + + \,|M|\, + + \rangle, & \phi_4 &= \langle + - \,|M|\, - + \rangle \;\propto\; -t \\
\phi_2 &= \langle + + \,|M|\, - - \rangle, & \phi_5 &= \langle + + \,|M|\, + - \rangle \;\propto\; \sqrt{-t} \\
\phi_3 &= \langle + - \,|M|\, + - \rangle, & \phi_6 &= \langle + + \,|M|\, - + \rangle \;\propto\; \sqrt{-t}
\end{aligned}
$$

Hadronic ϕ_1, ϕ_2, ϕ_3 are nonzero at $t = 0$, and $\phi_6 = -\phi_5$ for p p $\rightarrow$ p p

$$
\begin{aligned}
(k\sqrt{s}\,/\,2\pi)\,\sigma_{\mathrm{tot}} &= \mathrm{Im}\,[\phi_1(s,0) + \phi_3(s,0)] \\
\frac{2k^2 s}{\pi}\,\frac{d\sigma}{dt} &= |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2|\phi_5|^2 + 2|\phi_6|^2 \\
A_{\mathrm{N}}\,\frac{2k^2 s}{\pi}\,\frac{d\sigma}{dt} &= \mathrm{Im}\,[\,(\phi_1 + \phi_2 + \phi_3 - \phi_4)^* \phi_5\,] \\
A_{\mathrm{NN}}\,\frac{2k^2 s}{\pi}\,\frac{d\sigma}{dt} &= \mathrm{Re}\,[\,\phi_1^*\,\phi_2 - \phi_3^*\,\phi_4 - 2\phi_5^*\,\phi_6\,]
\end{aligned}
$$

Scattering of Identical and Non-identical Fermions

- For the elastic reactions $\mathrm{p\,p} \to \mathrm{p\,p}$ and $^3\mathrm{He}\,^3\mathrm{He} \to {}^3\mathrm{He}\,^3\mathrm{He}$, $\phi_6 = -\phi_5$
- For non-identical $\mathrm{p}\,^3\mathrm{He} \to \mathrm{p}\,^3\mathrm{He}$ or $^{13}\mathrm{C\,p} \to {}^{13}\mathrm{C\,p}$, in general, $\phi_6 \neq -\phi_5$

  NHB, Gotsman, Leader, Phys Rev D18 (1978) 694
- $^{12}\mathrm{C\,p} \to {}^{12}\mathrm{C\,p}$ or $^{12}\mathrm{C}\,^3\mathrm{He} \to {}^{12}\mathrm{C}\,^3\mathrm{He}$ would have just two amplitudes

Unequal mass CM momentum: $4\,k^2 \;=\; s - 2\,m^2 - 2\,\widetilde{m}^2 + \left(m^2 - \widetilde{m}^2\right)^2/s$

The proton carbon polarimeter requires calibration from a H-jet polarimeter operating at a 90 Hz rate to achieve a $\delta P \approx 2\%$ statistical accuracy for an 8-hour RHIC store. A. Poblaguev *et al.*, PoS PSTP **2017** (2018) 022

Interference Region Amplitudes for Identical Fermions

For t close to $t_{\mathrm{c}} = -8\,\pi Z^2\alpha/\sigma_{\mathrm{tot}}$, elastic amplitudes including Coulomb contributions and the phase, $\delta_{\mathrm{C}} = -\,Z^2\alpha\left(\,\ell\mathrm{n}\,|Bt/2\,+\,4t/\Lambda^2|\,+\,\gamma\,\right)$, are

$$\frac{4\,\pi}{k\sqrt{s}}\,\phi_1 \;=\; \sigma_{\mathrm{tot}}\,e^{Bt/2}\left[i\,+\,\rho\,-\,t_{\mathrm{c}}\,e^{i\,\delta_{\mathrm{C}}}\left(\frac{1}{t}\,+\,2\,b_1\,-\,B/2\right)\right]$$

$$\frac{4\,\pi}{k\sqrt{s}}\,\phi_2 \;=\; \sigma_{\mathrm{tot}}\,e^{Bt/2}\left[\,2\,r_2\,-\,t_{\mathrm{c}}\,e^{i\,\delta_{\mathrm{C}}}\left(\frac{\kappa}{2\,m_{\mathrm{p}}}\right)^2\,\right]$$

$$\frac{4\,\pi\,m_{\mathrm{p}}}{k\sqrt{-st}}\,\phi_5 \;=\; \sigma_{\mathrm{tot}}\,e^{Bt/2}\left[r_5\,-\,t_{\mathrm{c}}\,e^{i\,\delta_{\mathrm{C}}}\left(\frac{\kappa}{2\,t}\,-\,\frac{m^2}{s\,t}\right)\right]$$

The hadronic amplitude $\phi_4 \propto t$ is ignored, and also the difference $\phi_1 - \phi_3$.
C.M. momentum: $k = \left(s/4 - m^2\right)^{1/2}$. Dirac form factor: $F_1(t) \approx 1 + b_1 t$.

Differential cross section close to $t = t_{\rm c}$ with hadronic slope parameter B

$$\frac{16\pi}{\sigma_{\rm tot}^2}\frac{d\sigma}{dt}\,e^{-Bt} \;=\; \left(\frac{t_{\rm c}}{t}\right)^2 - \,2\,(\,\rho+\delta_{\rm C}+\epsilon\,)\,\frac{t_{\rm c}}{t} \,+\, 1 \,+\, \rho^2$$

Spin dependent observables in interference region of momentum transfer

$$\frac{m_{\rm p}\,A_{\rm N}}{\sqrt{-t}}\,\frac{8\,\pi}{\sigma_{\rm tot}^2}\,\frac{d\sigma}{dt}\,e^{-Bt} \;=\; \left[\frac{\kappa}{2}\,(1+{\rm Im}\,r_2-\delta_{\rm C}\,\rho)-{\rm Im}\,r_5+\delta_{\rm C}\,{\rm Re}\,r_5\right]\,\frac{t_{\rm c}}{t}$$
$$-\,(1+{\rm Im}\,r_2)\,{\rm Re}\,r_5 \,+\, (\rho+{\rm Re}\,r_2)\,{\rm Im}\,r_5$$

$$A_{\rm NN}\,\frac{8\,\pi}{\sigma_{\rm tot}^2}\,\frac{d\sigma}{dt}\,e^{-Bt} \;=\; -\left[{\rm Re}\,r_2 \,+\, \delta_{\rm C}\,{\rm Im}\,r_2\right]\frac{t_{\rm c}}{t} \,+\, \left(\kappa\,t_{\rm c}\,/m_{\rm p}^2\right){\rm Re}\,r_5$$
$$+\,{\rm Im}\,r_2 \,+\, \rho\,(\,{\rm Re}\,r_2 \,-\, \kappa^2 t_{\rm c}/4m_{\rm p}^2\,)$$

For protons, $\kappa = \mu_p - 1$; for He-3 (h), $\kappa = \mu_h/Z - m_{\rm p}/m_h$, with $Z = 2$.

A polarimeter requires a process with nonvanishing high energy polarization

- Spin one photon exchange suggests the Primakoff or a Coulomb effect
- Helium-3 scattering reaches about $-3\,\%$ asymmetry in the CNI region

A charge $Z'e$ scattering elastically off a spin half hadron of mass m, charge Ze, and magnetic moment μ has an asymmetry involving an interference

$$2\ \mathrm{Im}\left[\frac{Z\,Z'}{137\,t}\ +\ (\rho + i)\,\frac{\sigma_{\mathrm{tot}}}{8\,\pi}\right]^{*}\ \frac{\kappa\,\sqrt{-t}}{2\,m_{\mathrm{p}}}\ \left[\frac{Z\,Z'}{137\,t} + (\mathrm{Re}\,r_5 + i\ \mathrm{Im}\,r_5)\,\frac{\sigma_{\mathrm{tot}}}{8\,\pi}\right]$$

of helicity nonflip and flip amplitudes with electromagnetic and hadronic elements where σ_{tot} relates to the hadronic particles of charges Ze and $Z'e$.

Including the spin averaged denominator, the asymmetry is proportional to

$$A_{\mathrm{N}} \propto \frac{\sqrt{x}}{x^2+3}, \qquad x = \frac{t_{\mathrm{e}}}{t}, \qquad t_{\mathrm{e}} = -\frac{8\,\pi\,\sqrt{3}\,|\,Z\,Z'\,|}{137\,\sigma_{\mathrm{tot}}(s)} = \sqrt{3}\,t_{\mathrm{c}}$$

the extremum value of which occurs at $x = 1$, that is, at a transfer $t = t_{\mathrm{e}}$.
The optimum value of 3% to 4 % varies slowly with energy s as $1/\sqrt{\sigma_{\mathrm{tot}}(s)}$
It is either a maximum or minimum depending on the sign of a constant κ

$$A_{\mathrm{N}}^{\mathrm{opt}} = \frac{\kappa}{4m_{\mathrm{p}}}\sqrt{-3\,t_{\mathrm{e}}}\,, \qquad \kappa = \frac{\mu}{Z} - \frac{m_{\mathrm{p}}}{m}$$

The value of κ is 1.793 (anomalous μ) for protons and -1.398 for helions.
Hadronic helicity flip amplitudes and two photon exchange are ignored here.

Quantities related to ions A of charge Ze colliding elastically with polarised protons may be compared to those of the more familiar proton proton case

$$\frac{t_{\mathrm{e}}^{\mathrm{Ap}}}{t_{\mathrm{e}}^{\mathrm{pp}}} = \frac{Z\,\sigma_{\mathrm{tot}}^{\mathrm{pp}}}{\sigma_{\mathrm{tot}}^{\mathrm{Ap}}} \approx 0.74\,, \qquad \frac{A_{\mathrm{N}}^{\mathrm{Ap}}}{A_{\mathrm{N}}^{\mathrm{pp}}} = \left(\frac{t_{\mathrm{e}}^{\mathrm{Ap}}}{t_{\mathrm{e}}^{\mathrm{pp}}}\right)^{1/2} \approx 0.86$$

With distinct target fermions, by contrast, carbon helion and carbon proton elastic scattering have extremum momentum transfer and asymmetry ratios

$$\frac{t_{\mathrm{e}}^{\mathrm{Ch}}}{t_{\mathrm{e}}^{\mathrm{Cp}}} = \frac{2\,\sigma_{\mathrm{tot}}^{\mathrm{Cp}}}{\sigma_{\mathrm{tot}}^{\mathrm{Ch}}} \approx 1.0\,, \qquad \frac{A_{\mathrm{N}}^{\mathrm{Ch}}}{A_{\mathrm{N}}^{\mathrm{Cp}}} = \frac{\kappa_{\mathrm{h}}}{\kappa_{\mathrm{p}}}\left(\frac{t_{\mathrm{e}}^{\mathrm{Ah}}}{t_{\mathrm{e}}^{\mathrm{Ap}}}\right)^{1/2} \approx -0.78$$

The same would be approximately true if the projectile carbon (C) here were replaced throughout by another ion, including a proton or a helion ^{3}He

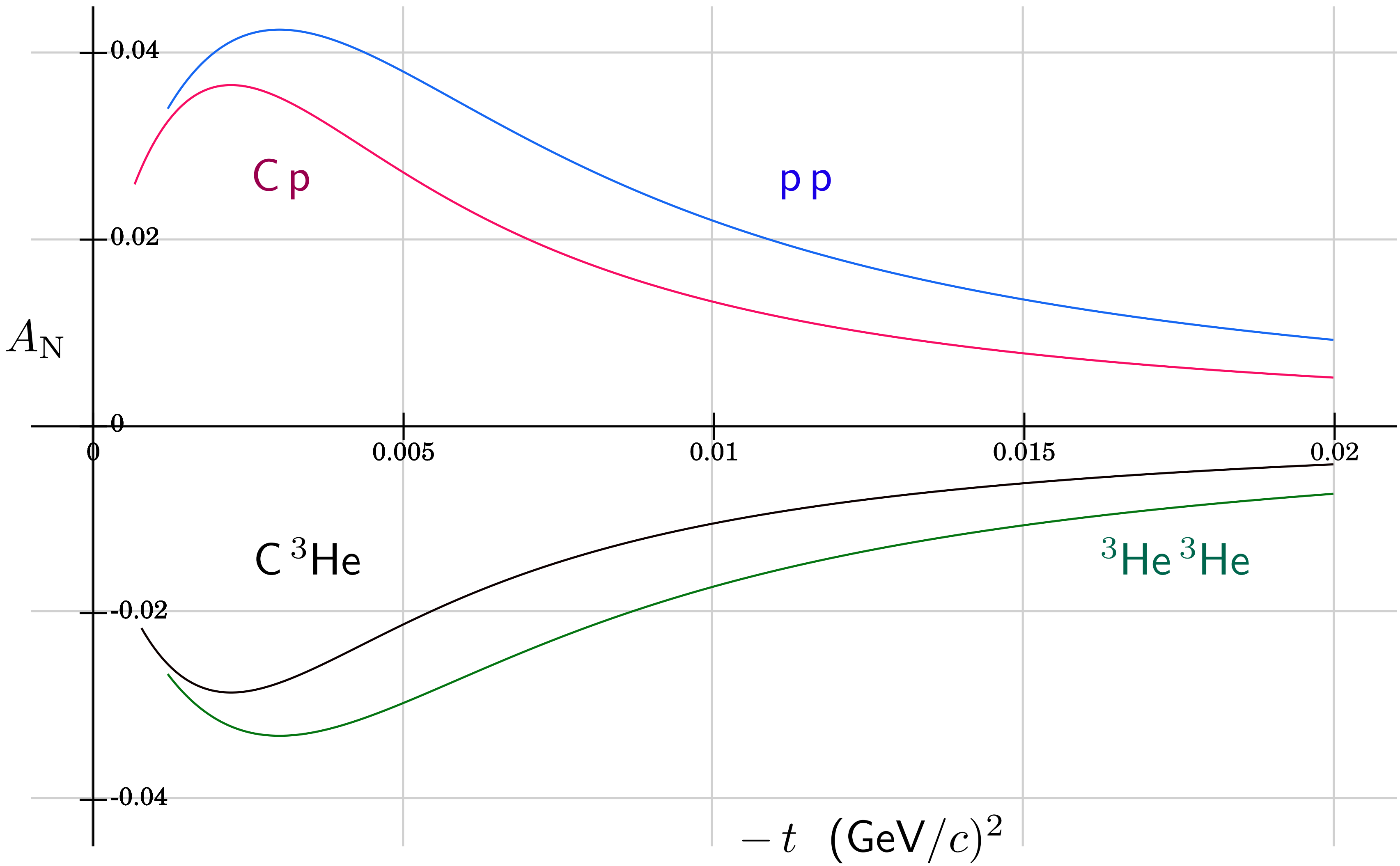

Figure 1: Analyzing power A_{N} versus invariant momentum transfer (-t) in (GeV/c)^2 for (1) p p and p h scattering, (2) C p scattering, (3) C h scattering, (4) h h and p h scattering

The extremum value of t has first order corrections in the Coulomb phase $\delta_{\rm C}$, the hadronic non-flip real part ratio ρ, and the helicity-flip ratio r_5

$$t_{\rm e}\ :\quad 1\ -\ (\rho+\delta_{\rm C})/\sqrt{3}\ -\ ({\rm Re}\,r_5\ -\ \rho\ {\rm Im}\,r_5)\,4/\sqrt{3}$$

Another factor with small items δ, ρ, r_5, multiplies the extremum of $A_{\rm N}$

$$A_{\rm N}\ :\quad 1\ +\ (\rho+\delta_{\rm C})\sqrt{3}\,/\,2\ -\ (\sqrt{3}\,{\rm Re}\,r_5\ +\ {\rm Im}\,r_5)$$

Polarised proton nucleus scattering has been studied over a range of momentum transfers Kopeliovich and Trueman, Phys Rev D **64** (2001) 034004

Polarised proton deuteron, $p^\uparrow$Al, and $p^\uparrow$Au elastic scattering has been measured at 100 GeV/n Poblaguev et al., SPIN 2016 Proceedings

Hadronic spin flip and Coulomb phase effects have been treated in detail in

NB, Kopeliovich, Leader, Soffer, Trueman, Phys Rev D59 (1999) 114010

The pp2pp experiment at STAR (RHIC BNL) has shown that the elastic pp

- hadronic single helicity-flip amplitude is small at $\sqrt{s} = 200$ GeV

  L. Adamczyk *et al.* [STAR Collaboration], Phys Lett B 719, 62 (2013)

The acceleration of Helium-3 nuclei to high energy has been discussed.

W. W. MacKay, AIP Conference Proceedings 980, 191 (2008)

Helium-3 ions have been accelerated in the AGS at BNL by Haixin Huang.
The Helium-3 carbon cross section at the AGS appears to be twice that for proton carbon scattering.

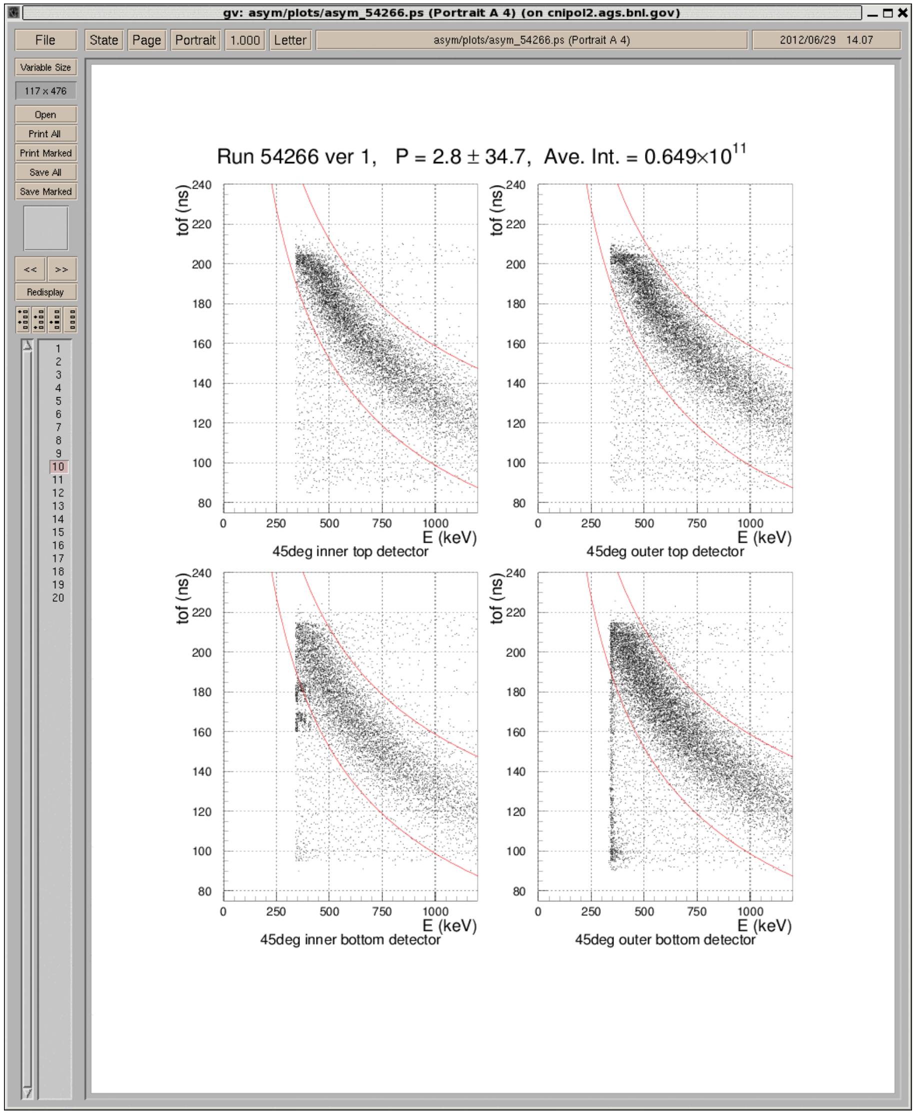

Figure 2: Time of flight of carbon recoils (on y-axis) versus the recoil kinetic energy of Helium-3 (on x-axis) as measured at the AGS. The 3He-C events are double those of p-C.

CONCLUSIONS

Probing the spin structure of hadrons increases an understanding of QCD

Measurements using polarised up and down quarks have great potential

Proton polarimetry is mature now and polarised ^{3}He may be forthcoming

The p ^{3}He$\uparrow$ analyzing power is $\approx -78\%$ of A_{N} for p p$\uparrow$ in the CNI region

CNI maximum: $A_{\mathrm{N}}(\,\mathrm{Ion\,p}\!\uparrow) \,/ A_{\mathrm{N}}(\mathrm{p\,p}\!\uparrow) \approx [\, Z_{\mathrm{Ion}}\, \sigma_{\mathrm{tot}}(\mathrm{p\,p}) \,/\, \sigma_{\mathrm{tot}}(\,\mathrm{Ion\,p})\,]^{1/2}$

Improved rate for ions by a factor $Z_{\mathrm{Ion}}^{1/2}$ is offset by reduced ion luminosity

Polarimetry at 7 TeV may require lab recoil angles from 87.14° to 88.28°.