Spin Asymmetry for Proton Deuteron Collisions at Forward Angles

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Abstract. The spin asymmetries for proton deuteron elastic scattering are studied at small angles in the context of understanding spin dynamics at high energy for the purposes of measuring the polarization of proton and deuteron beams. The effects of hadronic spin dependence, dispersive spin independent amplitudes, and the Coulomb phase are addressed in particular. Electromagnetic helicity amplitudes for proton deuteron collisions resulting from single photon exchange are presented, those prominent at low momentum transfer and high energy being highlighted. The character of the maximum analyzing power for colliding polarized protons, deuterons, and helium-3 is discussed, focusing on the dependence upon spin and phases that is important for polarimetry.

INTRODUCTION

In addition to the information on hadronic spin structure resulting from the availability of incident protons of known polarization there is a need to study other elememts of the isospin sector [1] using effective polarized neutrons at high energy. Polarized deuterons and helium-3 nuclei provide one source of such neutrons and an analysis of their electromagnetic and hadronic dynamics in collisions at small momentum transfers can assist the evaluation of their level of polarization.

The elastic scattering of spin polarized protons and deuterons is discussed here in the context of understanding its enriched spin dependence and of seeking a polarimeter for high energy deuterons. A comparison with the case of helium-3 of charge $Z = 2$ illuminates the discussion of the search for high energy neutrons of detectable spin polarization at squared momentum transfers in the interference region of [2]

$$-t_c = 8\pi Z\alpha/\sigma_{\text{tot}}\,. \tag{1}$$

Small angle proton carbon scattering has been used successfully [3] to evaluate high energy spin dependence [4] in the context of polarimetry. One of the significant results emerging from this study is that the hadronic single helicity flip amplitude appears to remain non-zero in the region of interference over a proton laboratory energy range extending to 100 GeV/c. There are hints of this in the proton proton case also [5].

With a deuteron spin of 1, the elastic scattering of protons on deuterons is another fermion boson process with rich spin properties. Elastic collisions at interference angles offer opportunities for probing a number of hadronic p d spin dependent amplitudes. Their rôle in deuteron relative polarimetry is highlighted.

SPIN 1/2 – 1 AMPLITUDES

The 36 helicity amplitudes of proton deuteron elastic scattering reduce to 12 independent ones under time reversal and parity invariance [6]. Of the hadronic amplitudes

$$H_i(\lambda_p', \lambda_d' \mid \lambda_p, \lambda_d), \quad i \in \{1, 2, \dots, 12\}, \quad \lambda_p \in \{+,-\}, \quad \lambda_d \in \{+,0,-\}$$

- four have imaginary parts relating to spin-dependent total cross sections

$$H_1(++|++), \quad H_2(++|-0), \quad H_3(+0|+0), \quad H_4(+-|+-)$$

- five amplitudes have the kinematic $\sqrt{-t}$ single helicity flip dependence as factor

$$H_5(++|+0),\ H_6(++|-+),\ H_7(++|--),\ H_8(+0\,|+-),\ H_9(+0\,|-0)$$

- two have a $(-t)$ factor and the twelfth amplitude a $-t\sqrt{-t}$ behavior near $t=0$

$$H_{10}(++|+-), \quad H_{11}(+0\,|-+); \qquad H_{12}(+-|-+).$$

In the reduction from 36 to 12 amplitudes, the multiplicity of the six amplitudes $H_1, H_3, H_4, H_7, H_9, H_{12}$ is two while that of the other six $H_2, H_5, H_6, H_8, H_{10}, H_{11}$ is four.

Coulomb amplitudes

One photon exchange helicity amplitudes have been calculated [7] in terms of the Dirac and Pauli proton form factors F_1, F_2, and deuteron form factors, F_1^{d}, F_2^{d}, G_1^{d}, which often appear in the linear combinations [8]

$$\begin{aligned} G_0^d &= F_1^d - \left[F_1^d + F_2^d\left(1-t/4M^2\right) - G_1^d\right] t/6M^2 \\ G_2^d &= F_1^d + F_2^d\left(1-t/4M^2\right) - G_1^d \end{aligned} \tag{2}$$

that have normalizations $G_0^d(0)=1,\ G_1^d(0)=\mu_d,$ and $G_2^d(0)=Q$, where the magnetic moment μ_d is in $e/2M$ units and the quadrupole moment Q is in units of e/M^2, the deuteron mass being $M = 1889.260$ MeV/c^2. In the case of electron deuteron scattering where $F_1 = 1$ and $F_2 = 0$ the amplitudes correctly reproduce the high energy unpolarized differential cross section for $e\,d \to e\,d$ collisions at center of mass scattering angle θ [9]

$$\frac{d\sigma}{dt} = 4\pi\left(\frac{\alpha}{t}\right)^2 \left\{ G_0^2 - \frac{t}{6M^2}\left[1+2\left(1-\frac{t}{4M^2}\right)\tan^2\frac{\theta}{2}\right] G_1^2 + \frac{t^2}{18M^2}\, G_2^2 \right\}. \tag{3}$$

Single photon exchange contributions to the $p\,d \to p\,d$ electromagnetic amplitudes have the following approximate form at asymptotic energies and low scattering angles

$$H_i^{\mathrm{em}}(+j\,|+j) = \frac{\alpha\, s}{t}\, F_1(t)\, F_1^{\mathrm{d}}(t), \quad i = 1, 3, 4 \tag{4}$$

in the helicity nonflip case for deuteron helicities $j \in \{+, 0, -\}$. The amplitude in which the proton of anomalous magnetic moment $\kappa_p = \mu_p - 1$, and mass m, flips its helicity is

$$H_6^{\mathrm{em}}(+\,+\,|\,-\,+) = \frac{\alpha\, s}{\sqrt{-t}}\, \frac{1}{2m}\, \kappa_p\, F_2(t)\, F_1^{\mathrm{d}}(t)\,. \tag{5}$$

The two deuteron single helicity flip electromagnetic amplitudes are equal at large s

$$H_5^{\mathrm{em}} = H_8^{\mathrm{em}} = \frac{\alpha\, s}{\sqrt{-2t}}\, F_1(t)\, \frac{1}{2M} \left[G_1^{\mathrm{d}}(t) - 2F_1^{\mathrm{d}}(t) \right] \tag{6}$$

and the remaining photon exchange amplitudes, H_2^{em}, H_7^{em}, H_9^{em}, H_{10}^{em}, H_{11}^{em}, H_{12}^{em}, are negligible asymptotically at forward angles.

ASYMMETRY EXTREMA

The analyzing power for polarized protons scattering on deuterons at electromagnetic interference involves, in particular, the following significant amplitudes

$$A_N = \frac{3\,\mathrm{Im}\left[H_6^{\,*}\,(H_1 + H_4) + \cdots\right]}{|H_1|^2 + 2\,|H_2|^2 + |H_3|^2 + |H_4|^2 + 2\,|H_5|^2 + 2\,|H_6|^2 + |H_7|^2 + 2\,|H_8|^2 + \cdots} \tag{7}$$

where each amplitude includes an electromagnetic contribution $H_i + e^{i\delta}\, H_i^{\mathrm{em}}$ involving spin 1/2 and 1 currents, with $\delta \approx 0.02$ as a small Coulomb phase. A factor of 1/3 appears in the averaged differential cross section due to the spin one nature of the deuteron. The analysis for polarized proton deuteron collisions follows that of the proton proton case [5, 10].

The analyzing power for high energy polarized protons (or indeed other polarized light fermion nuclei with charge Z like helium-3) colliding with deuterons with hadronic slope b in the interference region $-t_c = 8\pi Z\alpha/\sigma_{\mathrm{tot}}$ is given approximately by

$$\frac{mA_N}{\sqrt{-t}}\, \frac{16\pi}{\sigma_{\mathrm{tot}}^2}\, \frac{d\sigma}{dt}\, e^{-bt} \;\; = \;\; (\kappa - 2\,\mathrm{Im}\, r)\, \frac{t_c}{t} - 2\,\mathrm{Re}\, r + 2\rho\, \mathrm{Im}\, r \tag{8}$$

in which, neglecting the ratio of hadronic and electromagnetic form factors and other nuclear effects [11], the unpolarized differential cross section is

$$\frac{16\pi}{\sigma_{\mathrm{tot}}^2}\, \frac{d\sigma}{dt}\, e^{-bt} \;\; = \;\; \left(\frac{t_c}{t}\right)^2 - 2(\rho + \delta)\frac{t_c}{t} + \left(1 + \rho^2\right)\left(1 + \beta^2\right) \tag{9}$$

with ratio $\rho = \mathrm{Re}\, H_+/\mathrm{Im}\, H_+$ where the hadronic H_+ is an average of H_1, H_3, and H_4. Spin dependent hadronic terms present in the forward direction are incorporated in $\beta^2 =$

$$\frac{|H_1 - H_3 + H_4|^2 + |H_1 + H_3 - H_4|^2 + |H_1 - H_3 - H_4|^2 - |H_1|^2 - |H_3|^2 - |H_4|^2 + 6|H_2|^2}{|H_1 + H_3 + H_4|^2}$$

and β here is zero in the case where H_1, H_3, and H_4 are all equal, and $H_2 = 0$. Given that the hadronic single helicity flip amplitude seems to persist at high energies in proton carbon scattering [3] it would be most interesting to determine if any of the proton deuteron amplitudes contributing to β, or any of the kinematically scaled hadronic helicity flip amplitudes defined by [12]

$$r_i = m(-t)^{-1/2} H_i/\mathrm{Im} H_+ , \quad i = 5, 6, 8 , \tag{10}$$

prevail in the asymptotic energy region also. The behavior of r_6 from $\mathrm{p}\!\uparrow \mathrm{d} \to \mathrm{p}\!\uparrow \mathrm{d}$, and of r_5 and r_8 from $\mathrm{d}\!\uparrow \mathrm{p} \to \mathrm{d}\!\uparrow \mathrm{p}$ at large s would be of considerable interest. The evaluation of such terms would facilitate the understanding of high energy proton deuteron spin dynamics. With its low magnetic moment of 0.85744 nuclear magnetons the extremal analyzing power for polarized deuteron scattering is very much less than that of the proton case and would require increased running time for comparable accuracy.

POLARIZED HELIUM-3

The study of another method for providing polarized neutons is instructive. Consider the elastic collision of a spin half particle of mass m' and charge Z' with a particle of charge Z and suppose that ζ is the sign of $Z'Z$. Magnetic moments μ provide $\kappa/m = \mu/m - Z'/m'$, when given in nuclear magneton units. In the absence of hadronic spin dependence, the single spin asymmetry for nuclear size effects $b < -1/t_c$ in the electromagnetic interference region has an extremum of

$$A_{\mathrm{N}}^{\mathrm{e}} = \frac{\kappa\zeta}{4m}\sqrt{-3t_{\mathrm{e}}}\,, \quad \text{at} \quad -t_e = 8\pi\sqrt{3}\,\frac{|Z'Z|\alpha}{\sigma_{\mathrm{tot}}} \tag{11}$$

where for positive values of $\kappa\zeta$ the extremum is a maximum. In the proton case, for example, $\kappa = 1.79285$ nuclear magnetons, giving a maximum when scattering on a positive target. A helium-3 nucleus of charge $Z = 2$, mass $m' = 2808.392$ MeV/c^2, and $\kappa = -2.79569$ scattering on the same positive target would lead to a negative value of the analyzing power of about $-90\,\%$ times that of the proton case, assuming that the helium-3 total cross section is three times that of the proton total cross section at a corresponding energy. As in the proton case, the presence of high energy hadronic spin dependence suggests that only relative polarimetry is possible for helium-3 processes.

CONCLUSIONS

Polarimetry for high energy protons is being secured using the asymmetry in the elastic reaction $\mathrm{pC} \to \mathrm{pC}$. A study of the electromagnetic amplitudes for $\mathrm{pd} \to \mathrm{pd}$ reveals that such collisions may be used for either proton or deuteron polarization determination, but the small value of the magnetic moment of the deuteron hinders the evaluation of the spin polarization of the deuteron, in practice.

Another source of polarized neutron is provided by helium-3 nuclei. The minimum analyzing power for helium-3 elastic collisions at interference angles is expected to be about $-90\,\%$ of that of the proton case at similar energies due to the relatively large and negative magnetic moment of helium-3. For scattering on a positively charged target the helium-3 analyzing power is negative in the interference region.

A study of the spin averaged differential cross section over a range of energies at low momentum transfers enables a test of causality by way of the analytic properties of spin independent amplitudes. The further study of asymmetries in the elastic process at forward angles suggests that the evaluation of dispersion integrals for spin dependent scattering amplitudes may eventually be within reach.

The analysis of the many helicity amplitudes related to the peripheral elastic processes of a number of particles with spin in the electromagnetic-hadronic interference region would assist the measurement of particle polarization and provide a deepening understanding of high energy spin dynamics.

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