MA22S4 - CLASSICAL MECHANICS
ASSIGNMENT 3

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RULES

The deadline for submission is Thursday, April 5th at 10am. Late submissions without prior arrangement or a valid explanation will result in reduced marks. Late submissions after Wednesday 4pm in the following week (April 10th) will not be accepted, as the solutions will be published online after this final submission deadline to facilitate the exam preparations.

QUESTIONS

1. Consider a particle of unit mass moving along the x-axis with the potential energy function

\[ V(x) = \frac{1}{c} (x^2 - a^2)^2, \]

where \( a, c \) are both positive constants.

(a) Find the points of equilibrium for a particle moving under this potential. Determine if those are points of stable or unstable equilibrium. [20p]

(b) Sketch the potential as a function of \( x \). [10p]

(c) If there exists a point of stable equilibrium, find the frequency \( \omega \) of small oscillations around this point. [20p]

(d) What is the minimum amount of energy we need to give to the particle when sitting at \( x = 0 \), so that it escapes to infinity? [10p]

2. The force \( \vec{F} \) is given by

\[ \vec{F} = z^3 \hat{i} + 2y^2 \hat{j} + 3xz^2 \hat{k}, \]

where \( \hat{i}, \hat{j}, \hat{k} \) denote unit vectors in the direction of x-, y and z-axis, respectively.

(a) Is the force \( \vec{F} \) conservative? [10p]
(b) Find the work done by the force $\vec{F}$ when moving the particle of unit mass from point $P(0, 0, 0)$ to point $Q(0, \frac{\pi}{2}, 0)$ along the trajectory $r = \frac{\pi}{2} y \hat{j}$.

(c) Find the work done by a force moving a particle between the same points $P$ and $Q$ along a trajectory given by:

\[ x = 2\sin(2t), \quad y = t, \quad z = \cos^2(2t) - 1. \]