2. \[ \vec{F} = 2z \hat{i} + 2y^2 \hat{j} + 3x^2 \hat{k} \]

a) \( \vec{F} \) - conservative?

\[ \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2z^3 & 2y^2 & 3x^2 \end{vmatrix} = \hat{i} \left( 0 - 0 \right) + \hat{j} \left( 32^2 - 32^2 \right) + \hat{k} \left( 0 - 0 \right) = \vec{0} \]

\[ \Rightarrow \vec{F} \text{ is conservative.} \]

b) \( P(0, 0, 0); \quad Q(0, \frac{\pi}{2}, 0) \)

\[ \vec{F} = \overrightarrow{PQ} = \vec{C}: \quad x = 0; \quad y = \frac{\pi}{2} t; \quad z = 0 \]

\[ dr = \frac{\pi}{2} dt \hat{j} \]

\[ t = 0: \quad P(0, 0, 0) \]

\[ t = 1: \quad Q(0, \frac{\pi}{2}, 0) \]

\[ \vec{F} = 0 \hat{i} + 2(\frac{\pi}{2} t)^2 \hat{j} + 0 \hat{k} = \frac{\pi^2}{2} t^2 \hat{j} \]

\[ W = \int_{P}^{Q} \vec{F} \cdot dr = \int_{0}^{1} \frac{\pi^2}{2} t^2 dt = \frac{\pi^3}{4} t^2 \mid_{0}^{1} = \frac{\pi^3}{4} \]

\[ W = \int_{0}^{1} \frac{\pi^3}{4} t^2 dt = \frac{\pi^3}{4} \frac{t^3}{3} \mid_{0}^{1} = \frac{\pi^3}{12} \]
c) As the force is conservative, the work between the same points P, Q along any path is equal, therefore:

\[ W = \frac{\mathbf{F}^3}{12} \]

I leave to your discretion to give some points to those who attempted to solve the complicated integral explicitly, but did not arrive to the final result.

1. \( V(x) = ax^2 - bx^4 \); \( a, b \) - const > 0

a) Points of equilibrium are obtained from \( V'(x) = 0 \)

\[ V'(x) = 2ax - 4bx^3 \]

\[ V'(x) = x(2a - 4bx^2) \]

\[ V'(x) = 0 \quad \Rightarrow \quad x = 0 \quad \text{or} \quad x = \pm \sqrt{\frac{a}{2b}} \]

Whether the stationary point is stable/unstable Eq. point, we learn from \( V''(x) \):

\[ V''(x) = 2a - 12bx^2 \]

\[ \begin{cases} 
V''(x_0) = 2a > 0 : \text{STABLE} \\
V''(x_1) = V''(x_2) = 2a - 12b \cdot \frac{a}{2b} = -4a < 0 : \text{UNSTABLE} 
\end{cases} \]
1. b) To qualitatively sketch the potential:

- Evaluate $V(x)$ at the equilibrium points:
  \[ V(x_0) = 0 \]
  \[ V(x_{1,2}) = \alpha \cdot \frac{1}{2b} - b \cdot \frac{\alpha^2}{4b^2} = \frac{\alpha^2}{4b} > 0 \]

- Evaluate zeros of the potential:
  \[ V(x) = 0 \iff x^2 (a - bx^2) = 0 \implies x_0 = 0, x_{3,4} = \pm \sqrt{\frac{a}{b}} \]

- Behaviour of the potential at $\lim V(x)$:
  \[ V(x \to \pm \infty) = -\infty; \quad V(x \to -\infty) = -\infty \]

Point of stable Eq.: $x_0 = 0$

The period of small oscillations (derived in class) around the point of stable Eq. $x_0$ is

\[ T = 2\pi \sqrt{\frac{m}{V''(x_0)}} \]

For our particle of unit mass:

\[ T = \frac{2\pi}{\sqrt{V''(x_0)}} = \frac{2\pi}{\sqrt{2a}} \]
When the total energy is $0 < E < \frac{a^2}{4b}$ and as long as the particle is found in the potential "well" between $x_1$ and $x_2$, the motion is bounded.

For $E > \frac{a^2}{4b}$ the particle can escape to the infinity, as the energy is greater than the potential everywhere along the $x$-axis and the particle can move freely from $-\infty$ to $+\infty$ (unbounded motion).

(Any variation of the answer quoting "$E > \frac{a^2}{4b}$" is acceptable.)