Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

Trinity Term 2018

MA22S4: CLASSICAL MECHANICS

19/05/2018 Goldsmith Hall Main 9.30 — 11.30

Prof. Marina Krstic Marinkovic

Instructions to Candidates:

Credit will be given for the best 3 questions answered. All questions carry equal marks. The marks for each part of a question are indicated in square brackets at the begining of the part.

Materials Permitted for this Examination:

Formulae and Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination — please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. A particle of unit mass moves along a trajectory

$$r(\theta)=\begin{cases} a\ \cos\theta, & \theta\in\left(0,\frac{\pi}{2}\right), \text{and } \theta\in\left(\frac{3\pi}{2},2\pi\right) \\ -a\ \cos\theta, & \theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right) \end{cases}$$

expressed in plane polar coordinates. The angle $\theta(t)$ changes with time according to the equation $\theta=\omega t$. Here a,ω are positive constants independent of time.

   (a) [**10 marks**] Compute the transverse acceleration of the particle.

   (b) [**10 marks**] Find the force acting on a particle and express it in terms of the polar coordinates on the plane.

   (c) [**5 marks**] Show that the magnitude of the total force acting on a particle is constant.

2. A planet moves around its sun in an elliptic orbit with given eccentricity e and semilatus rectum l. A gravitational mass of the sun is defined as $\tilde{\mu}=GM_S$.

   (a) [**10marks**] Express the force vector acting on a planet in terms of the mass of the planet m_P, the angular momentum per unit mass h, semi-latus rectum l and the distance of the planet from its sun r. *[Hint: use the fact that for the central force the angular momentum per unit mass, h, is a conserved quantity: $h=r^2\dot{\theta}=const.$]*

   (b) [**15 marks**] Assume that at some point r_0 of its orbit, the speed of the planet is $v^2=\frac{\sqrt{3}}{2}\frac{\tilde{\mu}}{a}$, where a is the semi-major axis of the planet's trajectory. Find r_0 in terms of e, l and $\tilde{\mu}$.

3. The force $\vec{F}$ is given by

$$\vec{F}=x^2z\hat{\imath}+3y^3\hat{\jmath}+xz\hat{k},$$

where $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ denote unit vectors in the direction of x-, y and z-axis, respectively.

   (a) [**10 marks**] Is the force $\vec{F}$ conservative?

   (b) [**10 marks**] Find the work done by the force $\vec{F}$ when moving the particle of unit mass from point $P(1,0,\frac{\pi}{4})$ to point $Q(-1,0,-\frac{\pi}{4})$ along the trajectory $\vec{r}=\frac{\pi}{4}x\hat{k}$.

(c) [$\mathbf{5}$ **marks**] Find work done by a force moving a particle between P and Q along a trajectory given by: $x = \sin 2t$, $y = \cos 2t$, $z = t$.

4. (a) [$\mathbf{8}$ **marks**] Use Hamilton's principle to show that if $\mathcal{F}$ is any function of the generalized coordinates, then the Lagrangian functions $\mathcal{L}$ and $\mathcal{L} + \frac{d\mathcal{F}}{dt}$ yield the same equations of motion.

   (b) Consider a particle moving in a constant magnetic field. Its potential is given by

$$V(\vec{r}) = q\phi(\vec{r}) - q\dot{\vec{r}} \cdot \vec{A}(\vec{r})$$

   where $\vec{A}(\vec{r})$ is the $vector$ potential and $\phi(\vec{r})$ is the $scalar$ potential function.

   i. [$\mathbf{6}$ **marks**] Write the Lagrangian function and the Euler-Lagrange equations of motion.

   ii. [$\mathbf{6}$ **marks**] Find the generalized momenta and the Hamiltonian function.

   iii. [$\mathbf{5}$ **marks**] Is the total energy conserved? Explain why is this the case.