MA22S4 - Classical Mechanics
Assignment 4

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Rules

The deadline for submission is **Friday, April 20th at 2pm**. Please numerate your homework solutions, add your student ID to each page and submit via email to: mmarina@maths.tcd.ie, cc-ing obries39@tcd.ie, **specifying as a subject**: **[MA22S4 - Tutorial 4 submission]**. If you would prefer to hand in in person, please email us and we will make a special arrangement for your submission. Late submissions will not be accepted. The solutions will be published online after this final submission deadline, in order to help you prepare for the exam (taking place on 19/05/2018).

Attempt parts b) d) e),g), h) first. These already amount to 100p. Solving remaining parts is optional – it will bring you bonus points, which can make up for the ponts lost in tutorials 1-3.

Questions

1. Consider a particle moving in a constant magnetic field. We have learned in class that its potential can be written as

$$V(\vec{r}) = q\dot{\vec{r}} \cdot \vec{A}(\vec{r}) - q\phi(\vec{r})$$

where $\vec{A}(\vec{r})$ is the $vector$ potential, $\phi(\vec{r})$ is the $scalar$ potential and q is the charge of the particle.

(a) Knowing that $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$, prove that

$$\vec{F} = -q(\vec{E} + \dot{\vec{r}} \times \vec{B})$$

Hint 1: Prove that the above holds for the x-component of the force. The proof for y- and z-component is equivalent and there is no need to write those down.
Hint 2: Use the known relation between force and potential: $F_i = -\frac{\partial V}{\partial q_i} + \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_i}\right)$. [20]

(b) Write the Lagrangian function and the Euler-Lagrange equations of motion using 3-d Cartesian coordinates. *Hint: Since the magnetic field is given to be constant in this assignment, assuming e.g.* $\vec{B} = B\hat{z}$*.will simplify the calculation signifficantly. Use this to find explicit form of* $\vec{A}$*.* [20]

(c) Write the Lagrangian function and the Euler-Lagrange equations of motion using cylindrical coordinates. *Hint: Starting point same as in (a).* [20]

(d) Condider adding any function of the generalized coordinates $\mathcal{F}$ to the obtained lagrangian $\mathcal{L}$. Show that the Lagrangian functions $\mathcal{L}$ and $\mathcal{L}' = \mathcal{L} + \frac{d\mathcal{F}}{dt}$ yield the same equations of motion.
Hint 1: Use Hamilton's principle. [40]

(e) Find the generalized momenta and the Hamiltonian function using 3-d Cartesian coordinates. *Hint: Use same starting point and findings from part (b).* [20]

(f) Find the generalized momenta and the Hamiltonian function using cylindrical coordinates. *Hint: Use same starting point and findings from part (c).* [20]

(g) Compare the physical and generalized momenta for θ coordinate direction. [10]

(h) Is the total energy conserved? Explain why is this the case. [10]

(i) Assume that the electromagnetic field is homogenious and constant, in particular:

$$\vec{B} = B_z\hat{z} \text{ and } \vec{E} = E_z\hat{z}.$$

Find the trajectory of a particle of mass m and charge q for given boundary conditions:

$$\vec{r}(0) = 0 \text{ and } \dot{\vec{r}} = v_0\hat{x}.$$

[40]