MA22S4 - Classical Mechanics
Assignment 3

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Rules

The deadline for submission is **Friday, April 6th at 2pm**. You may hand in your write-up earlier in week, in the Wednesday's tutorials session (March 14th, 4pm) or during the Thursday lectures (April 5th, anytime). Late submissions without prior arrangement or a valid explanation will result in reduced marks. Late submissions after Wednesday 4pm in the following week (April 11th) will not be accepted, as the solutions will be published online after this final submission deadline.

Questions

1. Consider a particle of unit mass moving along the x-axis with the potential energy function
$$V(x) = ax^2 - bx^4,$$
where a, b are both positive constants.

   (a) Find the points of equilibrium for a particle moving under this potential. Determine if those are points of stable or unstable equilibrium. [20p]

   (b) Sketch the potential as a function of x. [10p]

   (c) If there exists a point of stable equilibrium, find the period of small oscillations around this point. [20p]

   (d) What is the minimum amount of energy we need to give to the particle when sitting at $x = 0$, so that it escapes to infinity? [10p]

2. The force $\vec{F}$ is given by
$$\vec{F} = z^3\hat{\imath} + 2y^2\hat{\jmath} + 3xz^2\hat{k},$$
where $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ denote unit vectors in the direction of x-, y and z-axis, respectively.

   (a) Is the force $\vec{F}$ conservative? [10p]

(b) Find the work done by the force $\vec{F}$ when moving the particle of unit mass from point $P(0,0,0)$ to point $Q(0,\frac{\pi}{2},0)$ along the trajectory $\vec{r} = \frac{\pi}{2}y\hat{j}$.

[10p]

(c) Find the work done by a force moving a particle between the same points P and Q along a trajectory given by:

$$x = 2\sin(2t), \quad y = t, \quad z = \cos^2(2t) - 1.$$

[10p]