IS2 In-class Homework Solution

Question
Let R be a relation on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ defined by the rule $x\ R\ y$ if $x - y$ is a multiple of 3.
Show that R is an equivalence relation and find the equivalence classes of R.
(multiples of 3 are $\{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots\}$)
Solution

$$\begin{aligned} R = \{ \quad & (1,7), (1,4), (2,5), (3,6), (4,7), (1,1), (2,2), (3,3), \\ & (4,4), (5,5), (6,6), (7,7), (4,1), (5,2), (6,3), (7,4), (7,1), \} \end{aligned}$$

- R is reflexive since $(x, x) \in R$ for all x in A.
  i.e. $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)$,
  $(6, 6)$ and $(7, 7)$ are in R.
- R is symmetric since $(x, y) \in R$ implies $(y, x) \in R$
  e.g. $(1, 7) \in R$ implies $(7, 1) \in R$.
- R is transitive since $(x, y) \in R$ and $(y, z) \in R$ implies (x, z) in R
  e.g. $(1, 7) \in R$ and $(7, 4) \in R$ implies $(1, 4) \in R$.
- Since R is reflexive, symmetric and transitive it is an equivalence relation.

$$\begin{aligned} E[1] &= \{y : (y, 1) \in R\} = \{1, 4, 7\} \\ E[2] &= \{y : (y, 2) \in R\} = \{2, 5\} \\ E[3] &= \{y : (y, 3) \in R\} = \{3, 6\} \\ E[4] &= \{y : (y, 4) \in R\} = \{1, 4, 7\} \\ E[5] &= \{y : (y, 5) \in R\} = \{2, 5\} \\ E[6] &= \{y : (y, 6) \in R\} = \{3, 6\} \\ E[7] &= \{y : (y, 7) \in R\} = \{1, 4, 7\} \end{aligned}$$

$\{\{1, 4, 7\}, \{2, 5\}, \{3, 6\}\}$ is the set of equivalence classes of R, these form a partition of A.