IS2 Homework I Solutions

Question 1
(i) p= " It is raining.", q = "I have an umbrella." $(p \wedge (\neg q)) \vee (\neg p)$
(ii) p="It is raining.", q="I can stay at home." $(\neg p) \rightarrow (\neg q)$

Question 2
(i)

p	q	$\neg q$	$(\neg 1) \wedge p$	$p \rightarrow ((\neg q) \wedge p)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

(ii)

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Question 3
p=" The two lines given are parallel."
q=" The two lines given have the same slope."
Statement is $p \rightarrow q$.

Converse $(q \rightarrow p)$
If the two lines given have the same slope, then they are parallel.

Inverse $(\neg p \rightarrow \neg q)$
If the two lines given are not parallel then they do not have the same slope.

Contrapositive $((\neg q) \rightarrow (\neg p))$
If the two lines given do not have the same slope then they are not parallel.

Negation $(\neg(p \rightarrow q) \equiv \neg(\neg p \vee q) \equiv p \wedge (\neg q))$
The two lines given are parallel and do not have the same slope.

Question 4
p="Sandra finishes her work", q="It snows", r="Sandra goes to the basket ball game."

$$p \rightarrow ((\neg q) \rightarrow r). \quad (\neg q). \quad \text{Therefore } (p \rightarrow r).$$

The following table shows this is a tautology, therefore the argument is valid.

p	q	r	$\neg q$	$(\neg q \to r)$	$p \to (\neg q \to r)$	$[p \to (\neg q \to r)] \wedge (\neg q)$
T	T	T	F	T	T	F
T	T	F	F	T	T	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	F
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	T	F	T	T

$p \to r$	$\{p \to (\neg q \to r) \wedge (\neg q)\} \to (p \to r)$
T	T
F	T
T	T
F	T
T	T
T	T
T	T
T	T

Question 5
(i) For every item in the whole world if it is a cat then it is not a dog.
(ii) There exists some item in the whole world that is not a dog and not a cat.

Question 6

$$\begin{aligned}
\neg(\forall x\ \forall y[P(x,y) \to Q(x,y)]) &\equiv \exists x\ (\neg(\forall y[P(x,y) \to Q(x,y)])) \\
&\equiv \exists x\ \exists y(\neg([P(x,y) \to Q(x,y)])) \\
&\equiv \exists x\ \exists y(\neg([(\neg P(x,y)) \vee Q(x,y)])) \\
&\equiv \exists x\ \exists y[P(x,y) \wedge (\neg Q(x,y))]
\end{aligned}$$

Question 7
(i) False (ii) False (iii) True (iv) False (v) True

Question 8
(i) $A \cup B = \{1, 2, 3, 5, 6, \{1, 2\}\}$
(ii) $A \cap C = \{1, 2, 8\}$
(iii) $B \cap \overline{C} = \emptyset$
(iv) $(A \cap \overline{B}) \cup C = \mathcal{E}$
(v) $\overline{(\overline{B} \cup \overline{C})} = \{3, 5, 8\}$

Question 9
7 students played no games.