BT 1120, Homework I

Question 1

$$\begin{aligned}
\frac{x+1}{x} - \frac{x+1}{x+2} &= \frac{(x+1)(x+2)-(x+1)(x)}{x(x+2)} \\
&= \frac{x^2+3x+2-x^2-x}{x(x+2)} \\
&= \frac{2x+2}{x(x+2)}
\end{aligned}$$

$$\begin{aligned}
\frac{\frac{x}{x+3}}{\frac{x^2+2x}{(x+3)(x+2)}} &= \frac{x}{x+3} \div \frac{x^2+2x}{(x+3)(x+2)} \\
&= \frac{x}{x+3} \times \frac{(x+3)(x+2)}{x^2+2x} \\
&= \frac{x(x+2)}{x+2} \\
&= 1
\end{aligned}$$

Question 2

(i) $y = -9x + 27$

(ii) $y = -\frac{17}{8}x + \frac{37}{8}$

Question 3

(a)Firstly

$$\begin{aligned}
y &= ax + b \\
y &= -7x + b
\end{aligned}$$

Then

$$\begin{aligned}
8 &= -7(3) + b \\
8 &= -21 + b \\
298 &= b
\end{aligned}$$

So

$$y = -7x + 29$$

(b) Firstly

$$\begin{aligned} y &= ax + b \\ 17 &= 2a + b \end{aligned}$$

$$20 = 3a + b$$

So

$$\begin{aligned} 20 - 17 &= a \\ 3 &= a \end{aligned}$$

So the slope is 3.

Question 4

(a)

$$\begin{aligned} x - y &= -1 \\ -x + 2y &= 5 \\ y &= 4 \\ x &= y - 1 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

So $(3, 4)$ is the unique solution.

(b)

$$\begin{aligned} 2x + 2y = 4 &\Rightarrow x + y = 2 \\ -12x - 12y = -24 &\Rightarrow x + y = 2 \end{aligned}$$

The two lines are the same, so there are an infinite number of solutions.

(c)

$$\begin{aligned} 3x - 7y &= 2 \\ x + 4y &= 11 \end{aligned}$$

Multiplying the second equation by (-3) gives:

$$\begin{aligned} 3x - 7y &= 2 \\ -3x - 12y &= -33 \end{aligned}$$

So

$$\begin{aligned} -19y &= -31 \\ y &= \frac{31}{19} \end{aligned}$$

Then find x:

$$\begin{aligned} x &= 11 - 4y \\ x &= 11 - 4\frac{31}{19} \\ x &= \frac{85}{19} \\ x &= 4\frac{9}{19} \end{aligned}$$

So $(4\frac{9}{19}, 1\frac{12}{19})$ is the unique solution.
(d)

$$\begin{aligned} y &= x + 4 \\ x - y &= -4 \end{aligned}$$

$$\begin{aligned} -2x + 2y &= 10 \\ x - y &= -5 \end{aligned}$$

No solution, so the two lines are parallel.

Question 5

$$\begin{aligned} P + 2Q &= 60 \\ P - Q &= 30 \end{aligned}$$

Multiplying the second equation by (-1) gives:

$$\begin{aligned} P + 2Q &= 60 \\ -P + Q &= -30 \end{aligned}$$

So we get

$$\begin{aligned} 3Q &= 30 \\ Q &= 10 \end{aligned}$$

Hence

$$P = 30 + Q = 30 + 10 = 40$$

This means that the equilibrium price is 40 and the equilibrium quantity is 10.

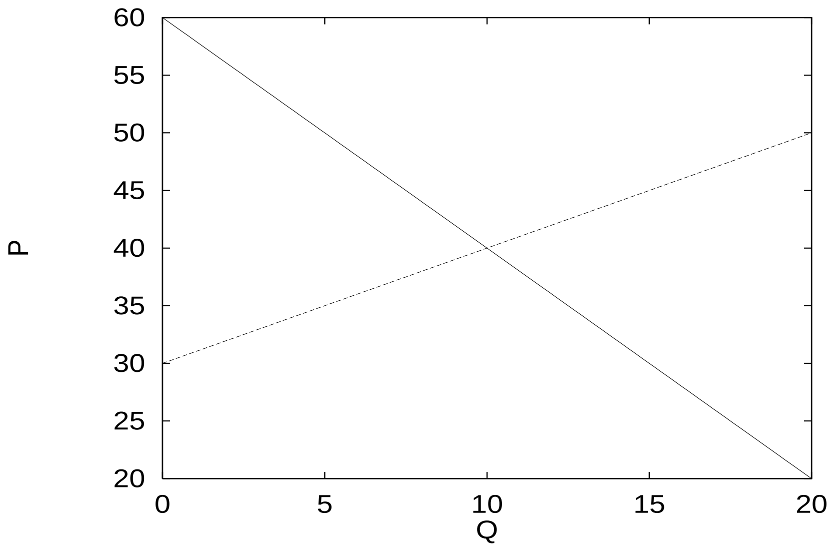

Figure 1: Plot of the two lines showing the equilibrium point

Question 6

Find the roots of the following quadratic functions:

(i) $y = x^2 - x - 6$

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{1 \pm \sqrt{1 + 24}}{2} \\ &= \frac{1 \pm \sqrt{25}}{2} \\ &= \frac{1 \pm 5}{2} \end{aligned}$$

So the roots are $x = 3$ and $x = -2$.

(ii) $y = 6x^2 + 24x - 24$ (10 marks)

$$x = \frac{-(24) \pm \sqrt{(24)^2 - 4(6)(-24)}}{2(6)}$$

$$
\begin{aligned}
&= \frac{-(24) \pm \sqrt{576 + 576}}{12} \\
&= \frac{-(24) \pm \sqrt{1152}}{12}
\end{aligned}
$$

So the roots are $x = 0.828$ and $x = -4.828$.

Question 7

(i) The coefficient of x^2 is $+1$, so the graph is "U" shaped.
$x = 0$ when $y = -15$ so $(0, -15)$ is on the graph.
$y = 0$ when $x^2 + 2x - 15 = 0$ solving this gives roots at $(-5, 0)$ and (3,0).

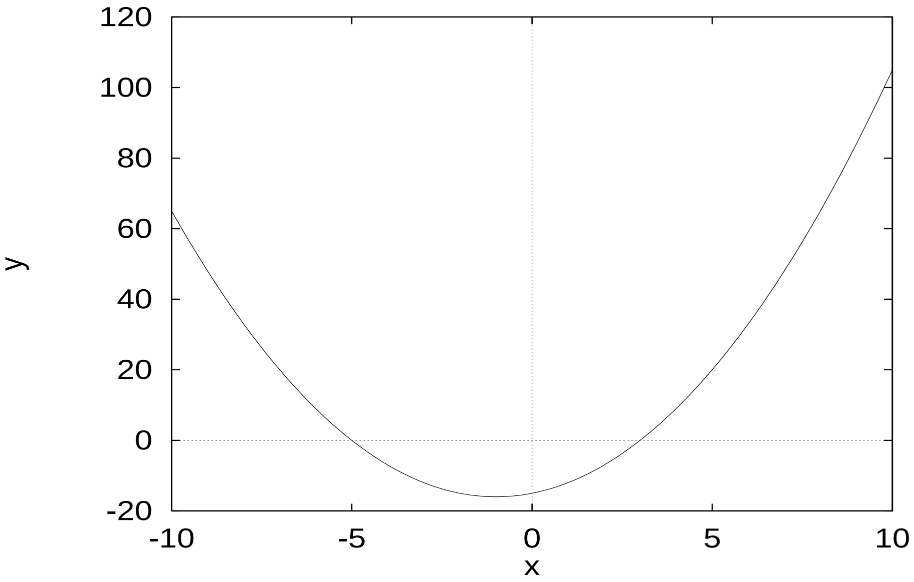

Figure 2: Plot of $y = x^2 + 2x - 15$

(ii) The coefficient of x^2 is -3, so the graph has an inverted "U" shape.
$x = 0$ when $y = 36$ so $(0, 36)$ is on the graph.
$y = 0$ when $-3x^2 - 3x + 36 = 0$ solving this gives roots at $(-4, 0)$ and (3,0).

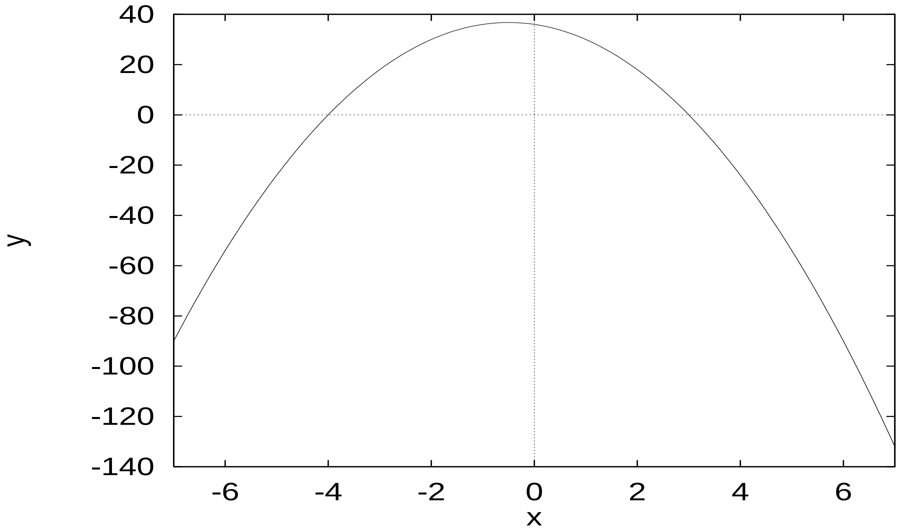

Figure 3: Plot of $y = -3x^2 - 3x + 36$

Question 8
(a)

$$\begin{aligned} \text{TR} &= P \cdot Q \\ &= (75 - 3Q)Q \\ &= 75Q - 3Q^2 \end{aligned}$$

(b) If TR=0 then

$$\begin{aligned} 75Q - 3Q^2 &= 0 \\ 3Q^2 - 75Q &= 0 \\ Q(3Q - 75) &= 0 \end{aligned}$$

So the roots are $Q = 0$ and $Q = 25$. (c) The maximum value occurs at $Q = 12\frac{1}{2}$ when

$$\begin{aligned} \text{TR} &= 75(12\frac{1}{2}) - 3(12\frac{1}{2})^2 \\ &= 468.75 \end{aligned}$$